Math 424 - Prof. Richard B. Goldstein - Chapter 3.4 HW – 7th Edition

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#1

(a) $\sum_{x=0}^{3}\sum_{y=0}^{3} f(x,y) = c\sum_{x=0}^{3}\sum_{y=0}^{3} xy = 36c = 1$. Hence $c = 1/36$.

(b) $\sum_x\sum_y f(x,y) = c\sum_x\sum_y |x-y| = 15c = 1$. Hence $c = 1/15$.

c|x – y|

	f(x,y)	x = -2	x = 0	x = 2
	-2	0	2c	4c
y	3	5c	3c	c

#2 $f(x,y) = \dfrac{x+y}{30}$ for $x = 0,1,2,3$ and $y = 0,1,2$

	f(x, y)	x = 0	x = 1	x = 2	x = 3
	0	0	1/30	2/30	3/30
y	1	1/30	2/30	3/30	4/30
	2	2/30	3/30	4/30	5/30

(a) $P(X \le 2, Y = 1) = f(0,1) + f(1,1) + f(2,1) = 1/30 + 2/30 + 3/30 = 1/5$.

(b) $P(X > 2, Y \le 1) = f(3,0) + f(3,1) = 3/30 + 4/30 = 7/30$.

(c) $P(X > Y) = f(1,0) + f(2,0) + f(3,0) + f(2,1) + f(3,1) + f(3,2)$
$= 1/30 + 2/30 + 3/30 + 3/30 + 4/30 + 5/30 = 3/5$.

(d) $P(X + Y = 4) = f(2,2) + f(3,1) = 4/30 + 4/30 = 4/15$.

#3

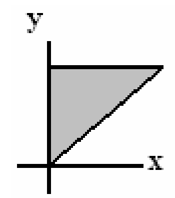
(a) We can select x oranges from 3, y apples from 2, and $4-x-y$ bananas from 3 in $\binom{3}{x}\binom{2}{y}\binom{3}{4-x-y}$ ways. A random selection of 4 pieces of fruit can be made in $\binom{8}{4}$ ways. Therefore,

$$f(x,y) = \frac{\binom{3}{x}\binom{2}{y}\binom{3}{4-x-y}}{\binom{8}{4}}, \qquad x = 0,1,2,3; \quad y = 0,1,2; \quad 1 \le x+y \le 4.$$

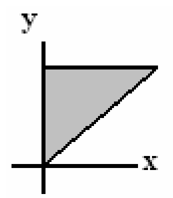
(b) $P[(X,Y) \in A] = P(X + Y \le 2) = f(1,0) + f(2,0) + f(0,1) + f(1,1) + f(0,2)$
$= 3/70 + 9/70 + 2/70 + 18/70 + 3/70 = 1/2$.

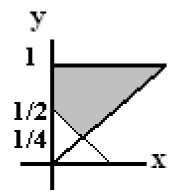
#7 (a) $P(0 \le X \le 1/2,\ 1/4 \le Y \le 1/2) = \int_0^{1/2} \int_{1/4}^{1/2} 4xy\ dy\ dx = 3/8 \int_0^{1/2} x\ dx = 3/64.$

(b) $P(X < Y) = \int_0^1 \int_0^y 4xy\ dx\ dy = 2\int_0^1 y^3\ dy = 1/2.$



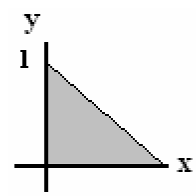
#9 $f(x,y) = \begin{cases} \frac{1}{y} & 0 < x < y < 1 \\ 0 & \text{elsewhere} \end{cases}$





$P(X + Y > 1/2) = 1 - P(X + Y < 1/2) = 1 - \int_0^{1/4} \int_x^{1/2-x} \frac{1}{y}\ dy\ dx$
$= 1 - \int_0^{1/4} \left[\ln\left(\frac{1}{2} - x\right) - \ln x\right]\ dx = 1 + \left[\left(\frac{1}{2} - x\right)\ln\left(\frac{1}{2} - x\right) - x\ln x\right]\Big|_0^{1/4}$
$= 1 + \frac{1}{4}\ln\left(\frac{1}{4}\right) = 0.6534.$

#22 $f(x,y) = \begin{cases} 6x & 0 < x < 1,\ 0 < y < 1 - x \\ 0 & \text{elsewhere} \end{cases}$



(a) $h(y) = 6\int_0^{1-y} x\ dx = 3(1-y)^2$, for $0 < y < 1$. Since $f(x|y) = \frac{f(x,y)}{h(y)} = \frac{2x}{(1-y)^2}$, for $0 < x < 1 - y$, involves the variable y, X and Y are not independent.

(b) $P(X > 0.3 \mid Y = 0.5) = 8\int_{0.3}^{0.5} x\ dx = 0.64.$

#24 $g(x) = 4\int_0^1 xy\ dy = 2x$, for $0 < x < 1$; $h(y) = 4\int_0^1 xy\ dx = 2y$, for $0 < y < 1$. Since $f(x,y) = g(x)h(y)$ for all (x,y), X and Y are independent.