

Math 424 – Prof. Richard B. Goldstein – Chapter 12 HW – 7<sup>th</sup> Edition

p 408 #2  $\hat{y} = -3.3727 + 0.0036x_1 + 0.9476x_2$ .

#6 (a)  $\hat{d} = 13.35875 - 0.33944v - 0.01183v^2$ .

(b)  $\hat{d} = 13.35875 - (-0.33944)(70) - (0.01183)(70)^2 = 47.54206$ .

p 419 #1  $s^2 = 0.16508$ .

#6 Using SAS output, we obtain

$0.4516 < \mu_{Y|x_1=900, x_2=1} < 1.2083$ , and  $-0.1640 < y_0 < 1.8239$ .

#10 The test statistic is  $t = \frac{0.00362}{0.000612} = 5.91$  with  $P$ -value = 0.0002. Reject  $H_0$  and claim that  $\beta_1 \neq 0$ .

Details

p 408

#2  $X'X = \begin{bmatrix} n & \sum x_{1i} & \sum x_{2i} \\ \sum x_{1i} & \sum (x_{1i})^2 & \sum x_{1i}x_{2i} \\ \sum x_{2i} & \sum x_{1i}x_{2i} & \sum (x_{2i})^2 \end{bmatrix} = \begin{bmatrix} 12 & 11280 & 8.12 \\ 11280 & 11,043,750 & 7632.8 \\ 8.12 & 7632.8 & 6.762 \end{bmatrix}$

$(X'X)^{-1} = \begin{bmatrix} 2.450262 & -0.0021337 & -0.53387 \\ -0.00213 & 2.26989E-06 & -3E-18 \\ -0.53387 & -3.5927E-18 & 0.788975 \end{bmatrix}$

$X'y = \begin{bmatrix} \sum y_i \\ \sum x_{1i}y_i \\ \sum x_{2i}y_i \end{bmatrix} = \begin{bmatrix} 8.019 \\ 9131.205 \\ 6.62724 \end{bmatrix}$

$b = (X'X)^{-1} X'y = \begin{bmatrix} -3.37267 \\ 0.003617 \\ 0.947599 \end{bmatrix}$

Excel formulas =minverse(array) and =mmult(array1,array2) are used by the following procedure: (1) highlight output cells, (2) press [F2], (3) enter the formula, (4) Ctrl+Shift+Enter

$$\#6 \quad X'X = \begin{bmatrix} n & \sum x_i & \sum x_i^2 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \end{bmatrix} = \begin{bmatrix} 6 & 435 & 35475 \\ 435 & 35475 & 3142875 \\ 35475 & 3142875 & 294421875 \end{bmatrix}$$

$$(X'X)^{-1} = \begin{bmatrix} 12.69735 & -0.37132275 & 0.002433862 \\ -0.371323 & 0.011378307 & -7.67196E-05 \\ 0.002434 & -7.672E-05 & 5.29101E-07 \end{bmatrix}$$

$$X'y = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \end{bmatrix} = \begin{bmatrix} 352 \\ 30935 \\ 2888725 \end{bmatrix}$$

$$b = (X'X)^{-1} X'y = \begin{bmatrix} 13.35873 \\ -0.33944 \\ 0.011825 \end{bmatrix}$$

p 419

$$\#2 \quad SSE = y'y - bX'y = 13.7452 - \begin{bmatrix} -3.3727 & 0.003617 & 0.9746 \end{bmatrix} \begin{bmatrix} 8.019 \\ 9131.2 \\ 6.62724 \end{bmatrix}$$

$$SSE = \sum (y_i - \hat{y}_i)^2 = 1.485727, \quad MSE = s^2 = SSE/9 = 0.16508 \quad s = 0.4063$$

$$\#6 \quad \hat{y} = -3.37267 + 0.003617(900) + 0.947599(1) = 0.8300$$

$$x'_0 C x_0 = \begin{bmatrix} 1 & 900 & 1 \end{bmatrix} (X'X)^{-1} \begin{bmatrix} 1 \\ 900 \\ 1 \end{bmatrix} = 0.16945 \quad t_{\alpha/2} = 2.2622$$

$$E = 2.2622(0.4063)\sqrt{0.16945} = 0.3784 \quad 0.8300 \pm 0.3784 \text{ or } [0.4516, 1.2084]$$

$$E = 2.2622(0.4063)\sqrt{1 + 0.16945} = 0.9940 \quad 0.8300 \pm 0.9940 \text{ or } [-0.1640, 1.8240]$$

$$\#10 \quad s\sqrt{C_{11}} = 0.4063\sqrt{2.26989 \times 10^{-6}} = 0.0006121$$

$$t = \frac{0.003617}{0.0006121} = 5.909 \quad p = \text{tdist}(5.909, 9, 2) = 0.000227$$