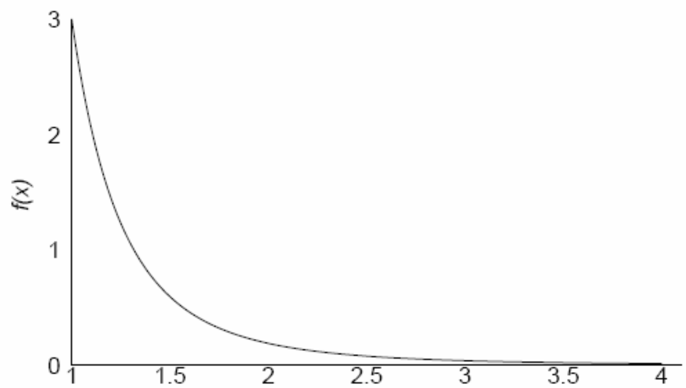
Math 423 – Prof. Richard B. Goldstein – Walpole – 8th ed – Chapter 4 Answers

4.3 $\mu = E(X) = (20)(1/5) + (25)(3/5) + (30)(1/5) = 25$ cents.

4.5 $\mu = E(X) = (0)(0.41) + (1)(0.37) + (2)(0.16) + (3)(0.05) + (4)(0.01) = 0.88.$

4.13 $E(X) = \frac{4}{\pi}\int_0^1 \frac{x}{1+x^2}\, dx = \frac{\ln 4}{\pi}.$

4.29 (a) The density function is shown next



(b) $\mu = E(X) = \int_1^\infty 3x^{-3}\, dx = \frac{3}{2}.$

4.55 $E(X) = (-3)(1/6) + (6)(1/2) + (9)(1/3) = 11/2,$
$E(X^2) = (-3)^2(1/6) + (6)^2(1/2) + (9)^2(1/3) = 93/2.$ So,
$E[(2X+1)^2] = 4E(X^2) + 4E(X) + 1 = (4)(93/2) + (4)(11/2) + 1 = 209.$

4.57 The equations $E[(X-1)^2] = 10$ and $E[(X-2)^2] = 6$ may be written in the form:

$$E(X^2) - 2E(X) = 9, \qquad E(X^2) - 4E(X) = 2.$$

Solving these two equations simultaneously we obtain

$$E(X) = 7/2, \quad \text{and} \quad E(X^2) = 16.$$

Hence $\mu = 7/2$ and $\sigma^2 = 16 - (7/2)^2 = 15/4.$

4.68 $\mu = E(X) = 6\int_0^1 x^2(1-x)\ dx = 0.5$, $E(X^2) = 6\int_0^1 x^3(1-x)\ dx = 0.3$, which imply $\sigma^2 = 0.3 - (0.5)^2 = 0.05$ and $\sigma = 0.2236$. Hence,

$$\begin{aligned} P(\mu - 2\sigma < X < \mu + 2\sigma) &= P(0.5 - 0.4472 < X < 0.5 + 0.4472) \\ &= P(0.0528 < X < 0.9472) = 6\int_{0.0528}^{0.9472} x(1-x)\ dx = 0.9839, \end{aligned}$$

compared to a probability of at least 0.75 given by Chebyshev's theorem.