Math 423 – **Prof. Richard B. Goldstein** – Walpole – 8th ed – Chapter 10 Answers

10.4 (a) $\alpha = P(X \le 5 \mid p = 0.6) + P(X \ge 13 \mid p = 0.6) = 0.0338 + (1 - 0.9729) = 0.0609.$

(b) $\beta = P(6 \le X \le 12 \mid p = 0.5) = 0.9963 - 0.1509 = 0.8454.$
$\beta = P(6 \le X \le 12 \mid p = 0.7) = 0.8732 - 0.0037 = 0.8695.$

(c) This test procedure is not good for detecting differences of 0.1 in p.

10.8 (a) $n = 12$, $p = 0.7$, and $\alpha = P(X > 11) = 0.0712 + 0.0138 = 0.0850.$

(b) $n = 12$, $p = 0.9$, and $\beta = P(X \le 10) = 0.3410.$

10.15 (a) $\mu = 200$, $n = 9$, $\sigma = 15$ and $\sigma_{\bar{X}} = \frac{15}{3} = 5$. So,

$$z_1 = \frac{191 - 200}{5} = -1.8, \quad \text{and} \quad z_2 = \frac{209 - 200}{5} = 1.8,$$

with $\alpha = 2P(Z < -1.8) = (2)(0.0359) = 0.0718.$

(b) If $\mu = 215$, then $z - 1 = \frac{191-215}{5} = -4.8$ and $z_2 = \frac{209-215}{5} = -1.2$, with

$$\beta = P(-4.8 < Z < -1.2) = 0.1151 - 0 = 0.1151.$$

10.21 The hypotheses are

$$H_0 : \mu = 40 \text{ months},$$
$$H_1 : \mu < 40 \text{ months}.$$

Now, $z = \frac{38-40}{5.8/\sqrt{64}} = -2.76$, and P-value$= P(Z < -2.76) = 0.0029$. Decision: reject H_0.

10.25 The hypotheses are

$$H_0 : \mu = 10,$$
$$H_1 : \mu \ne 10.$$

$\alpha = 0.01$ and $df = 9$.
Critical region: $t < -3.25$ or $t > 3.25$.
Computation: $t = \frac{10.06-10}{0.246/\sqrt{10}} = 0.77$.
Decision: Fail to reject H_0.

10.27 The hypotheses are

$$H_0 : \mu_1 = \mu_2,$$
$$H_1 : \mu_1 > \mu_2.$$

Since $s_p = \sqrt{\frac{(29)(10.5)^2+(29)(10.2)^2}{58}} = 10.35$, then

$$P\left[T > \frac{34.0}{10.35\sqrt{1/30 + 1/30}}\right] = P(Z > 12.72) \approx 0.$$

Hence, the conclusion is that running increases the mean RMR in older women.

10.55 The hypotheses are

$$H_0 : p = 0.40,$$
$$H_1 : p > 0.40.$$

Denote by X for those who choose lasagna.

$$P\text{-value} = P(X \geq 9 \mid p = 0.40) = 0.4044.$$

The claim that $p = 0.40$ is not refuted.

10.59 The hypotheses are

$$H_0 : p = 0.2,$$
$$H_1 : p < 0.2.$$

Then

$$P\text{-value} \approx P\left(Z < \frac{136 - (1000)(0.2)}{\sqrt{(1000)(0.2)(0.8)}}\right) = P(Z < -5.06) \approx 0.$$

Decision: Reject H_0; less than 1/5 of the homes in the city are heated by oil.

10.67 The hypotheses are

$$H_0 : \sigma^2 = 0.03,$$
$$H_1 : \sigma^2 \neq 0.03.$$

Computation: $\chi^2 = \frac{(9)(0.24585)^2}{0.03} = 18.13$. Since $0.025 < P(\chi^2 > 18.13) < 0.05$ with 9 degrees of freedom, $0.05 < P\text{-value} = 2P(\chi^2 > 18.13) < 0.10$.
Decision: Fail to reject H_0; the sample of 10 containers is not sufficient to show that σ^2 is not equal to 0.03.

10.73 The hypotheses are

$$H_0: \sigma_1^2 = \sigma_2^2,$$
$$H_1: \sigma_1^2 > \sigma_2^2.$$

Computation: $f = \frac{(6.1)^2}{(5.3)^2} = 1.33$. Since $f_{0.05}(10, 13) = 2.67 > 1.33$, we fail to reject H_0 at level $\alpha = 0.05$. So, the variability of the time to assemble the product is not significantly greater for men. On the other hand, if you use "=fdist(1.33,10,13)", you will obtain the P-value $= 0.3095$.

10.83 The hypotheses are

$$H_0: \text{Data follows the binomial distribution } b(y; 3, 1/4),$$

$$H_1: \text{Data does not follows the binomial distribution.}$$

$\alpha = 0.01$.
Computation: $b(0; 3, 1/4) = 27/64$, $b(1; 3, 1/4) = 27/64$, $b(2; 3, 1/4) = 9/64$, and $b(3; 3, 1/4) = 1/64$. Hence $e_1 = 27$, $e_2 = 27$, $e_3 = 9$ and $e_4 = 1$. Combining the last two classes together, we obtain

$$\chi^2 = \frac{(21-27)^2}{27} + \frac{(31-27)^2}{27} + \frac{(12-10)^2}{10} = 2.33.$$

Critical region: $\chi^2 > 9.210$ with 2 degrees of freedom.
Decision: Fail to reject H_0; the data is from a distribution not significantly different from $b(y; 3, 1/4)$.

10.90 The hypotheses are

H_0 : Presence or absence of hypertension is independent of smoking habits,

H_1 : Presence or absence of hypertension is not independent of smoking habits.

$\alpha = 0.05$.
Critical region: $\chi^2 > 5.991$ with 2 degrees of freedom.
Computation:

Observed and expected frequencies				
	Nonsmokers	Moderate Smokers	Heavy Smokers	Total
Hypertension	21 (33.4)	36 (30.0)	30 (23.6)	87
No Hypertension	48 (35.6)	26 (32.0)	19 (25.4)	93
Total	69	62	49	180

$$\chi^2 = \frac{(21-33.4)^2}{33.4} + \cdots + \frac{(19-25.4)^2}{25.4} = 14.60.$$

Decision: Reject H_0; presence or absence of hypertension and smoking habits are not independent.

10.96 The hypotheses are

H_0 : The proportions of widows and widowers are equal with respect to the different time period,

H_1 : The proportions of widows and widowers are not equal with respect to the different time period.

$\alpha = 0.05$.
Critical region: $\chi^2 > 5.991$ with 2 degrees of freedom.
Computation:

Observed and expected frequencies			
Years Lived	Widow	Widower	Total
Less than 5	25 (32)	39 (32)	64
5 to 10	42 (41)	40 (41)	82
More than 10	33 (26)	21 (26)	54
Total	100	100	200

$$\chi^2 = \frac{(25-32)^2}{32} + \frac{(39-32)^2}{32} + \cdots + \frac{(21-26)^2}{26} = 5.78.$$

Decision: Fail to reject H_0; the proportions of widows and widowers are equal with respect to the different time period.