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Chapter 5

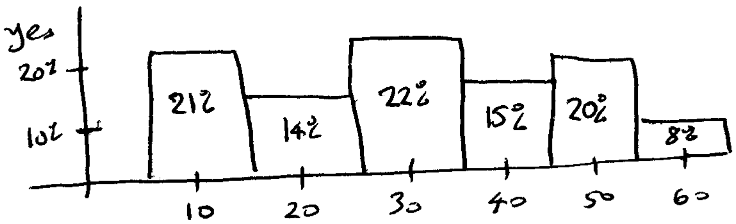
Sect 5.1 #7

Income Range	5–15	15–25	25–35	35–45	45–55	55+
Midpt	10	20	30	40	50	60
%	21%	14%	22%	15%	20%	8%

sum = 100% ✓

(a) yes

(b)



(c) $\mu = \sum x_i f_i$ or $\sum x_i p_i = 10(0.21) + 20(0.14) + 30(0.22) + 40(0.15) + 50(0.2) + 60(0.08)$
$= \underline{32.3}$

(d) $\sigma^2 = \sum x^2 p_i - \mu^2 = 10^2(0.21) + 20^2(0.14) + 30^2(0.22) + 40^2(0.15) + 50^2(0.2) + 60^2(0.08) - 32.3^2$
$= 1303 - 1043.29 = 259.71$

$\sigma = \sqrt{259.71} \approx \underline{16.12}$

Sect 5.2 #14

$p = 0.1 \quad n = 20$

(a) $P(X \geq 1) = 1 - P(X<1) = 1 - P(0) = 1 - 0.9^{20} = \underline{0.8784}$

(b) $P(X > 2) = 1 - P(X \leq 2) = 1 - P(0) - P(1) - P(2)$
$= 1 - 0.9^{20} - 20(0.1)(0.9)^{19} - 190(0.1)^2(0.9)^{18}$
$= 1 - 0.1216 - 0.2702 - 0.2852 = \underline{0.3230}$

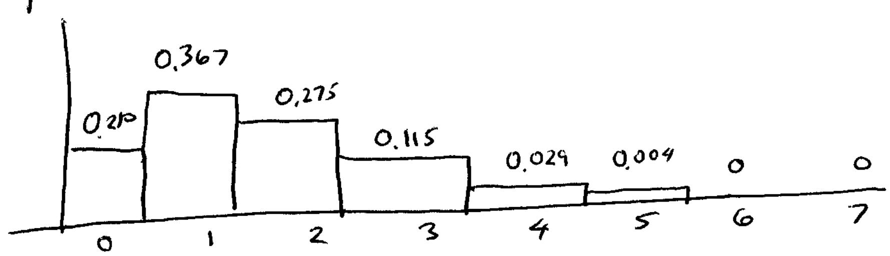
(c) $P(X=0) = 0.9^{20} = \underline{0.1216}$

(d) P(at least 18 $\underline{\underline{\text{not}}}$ too tight) ≡ P(0, 1 or 2 too tight)
$= P(0) + P(1) + P(2) = 0.9^{20} + 20(0.1)(0.9)^{17}$ $190(0.1)^2(0.9)^{18}$ $= 0.1216 + 0.2702 + 0.2852 = \underline{0.6770}$

Sect 5.3 #11

$p = 0.2$ illiterate $\quad n = 7$

(a)



(b) $\mu = np = 7(0.2) = \underline{\underline{1.4}}$ $\quad \sigma^2 = npq = 7(0.2)(0.8) = 1.12 \quad \sigma = \sqrt{1.12} = \underline{1.058}$

(c) $n = 12$ gives $P(r \geq 7) = 1 - 0.020 = \underline{0.980}$
(use $p = 0.8$ here)

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Sect 5.4

#6 $p = 0.57$

(a) $P(n) = 0.57(0.43)^{n-1}$

(b) $P(2) = 0.57(0.43) = 0.2451$

(c) $P(3) = 0.57(0.43)^2 = 0.1054$

(d) $\mu = \frac{1}{p} = \frac{1}{0.57} = 1.75 \approx 2$

#11 (a) $\lambda = 3(1.7) = 5.1$ per 30 minute period

(b) $P(r) = \frac{e^{-5.1}(5.1)^r}{r!}$

(c) $P(4) \simeq \underline{0.1719}$ $\quad P(5) \simeq \underline{0.1753}$ $\quad P(6) = \frac{e^{-5.1}(5.1)^6}{6!} \simeq \underline{0.1490}$

(d) $P(r<4) = P(0) + P(1) + P(2) + P(3)$
$= 0.0061 + 0.0311 + 0.0793 + 0.1348 = \underline{0.2513}$

#17 (a) small p any day, n large number of days

(b) $\lambda = 60\left(\frac{8}{275}\right) = \underline{1.75}$

$P(0) = \frac{e^{-1.75}\,1.75^0}{0!} = \underline{0.174}$, $P(1) = \underline{0.304}$, $P(r \geq 2) = 1 - 0.174 - 0.304 \simeq \underline{0.522}$

(c) $\lambda = 90\left(\frac{8}{275}\right) = 2.62$

$P(0) = e^{-2.62} = \underline{0.0728}$, $P(2) = \frac{e^{-2.62}(2.62)^2}{2!} = \underline{0.2499}$

$P(r \geq 3) = 1 - P(r<3) = 1 - P(0) - P(1) - P(2)$
$= 1 - 0.0728 - 0.1907 - 0.2499 = \underline{0.4866}$

Chapter 6

Sect 6.1

#7 (a) $P(x > 65'') = 0.50 = 50\%$

(b) $P(x < 65'') = 0.50 = 50\%$

(c) $P(62.5'' < x < 67.5'') = 68\%$ $\quad (2(0.34))$

(d) $P(60'' < x < 70'') = 95\%$ $\quad (2(0.475))$

Sect 6.2

#9 $\mu = 4.8$ $\quad \sigma = 0.3$ $\quad$ RBC $\frac{\text{millions}}{mm^3}$

(a) $P(4.5 < x) = P\left(z > \frac{4.5-4.8}{0.3}\right) = P(\underline{\underline{z > -1}})$

(b) $x < 4.2 \Rightarrow z < \frac{4.2-4.8}{0.2} = -2$ $\quad \underline{\underline{z < -2}}$

(c) $4.0 < x < 5.5 \Rightarrow \frac{4.0-4.8}{0.3} < z < \frac{5.5-4.8}{0.3} \Rightarrow \underline{\underline{-2.67 < z < 2.33}}$

Sect 6.2 #9 continued

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(d) $z < -1.44 \Rightarrow x < 4.8 - 1.44(0.3)$
$\underline{x < 4.368}$

(e) $1.28 < z \Rightarrow 4.8 + 1.28(0.3) < x \Rightarrow \underline{5.184 < x}$

(f) $-2.25 < z < -1.00 \Rightarrow 4.8 - 2.25(0.3) < x < 4.8 - 1.00(0.3)$
$\Rightarrow \underline{4.125 < x < 4.5}$

(g) $z = \dfrac{5.9 - 4.8}{0.3} = 3.67$ – would be unusually high

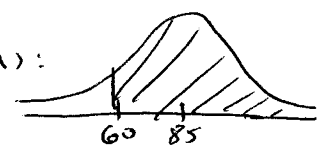
#19 $P\{z \geq -1.22\} = 1 - 0.1112 = \underline{0.8888}$

#25 $P\{0.32 \leq z \leq 1.92\} = 0.9726 - 0.6255 = \underline{0.3471}$

Sect 6.3

#25 $\mu = 85 \ \frac{mg}{dl}$ $\sigma = 25 \ \frac{mg}{dl}$

(a):



(a) $P(x > 60) = P\left(z > \dfrac{60-85}{25}\right) = P(z > -1) = 1 - 0.1587 = \underline{0.8413}$

(b) $P(x < 110) = P\left(z < \dfrac{110-85}{25}\right) = P(z < 1) = \underline{0.8413}$

(c) $P(60 < x < 110) = P(-1 < z < 1) = 0.8413 - 0.1587 = \underline{0.6826}$

(d) $P(x > 140) = P\left(z > \dfrac{140-85}{25}\right) = P(z > 2.2) = 1 - 0.9861 = \underline{0.0139}$

#31 $\mu = 8$ yrs. 95% 5 to 11 yrs. $\sigma \approx \dfrac{11-5}{4} = \underline{1.5 \text{ yrs.}}$

(a)

(b) $P(x > 5) = P\left(z > \dfrac{5-8}{1.5}\right) = P(z > -2) = 1 - 0.0228 = \underline{0.9772}$

(c) $P(x < 10) = P\left(z < \dfrac{10-8}{1.5}\right) = P(z < 1.33) = \underline{0.9082}$

(d) $P(z < -1.28) \cong 0.100 \Rightarrow x = 8 - 1.28(1.5) \approx \underline{6.1 \text{ yrs}}$

Sect 6.4

#7 $p = 0.035$ $n = 753$ $\mu = np = 26.355$ (75) $\sigma = \sqrt{npq} = 5.043$

(a) $P(X \geq 15) = P\left(z \geq \dfrac{14.5 - 26.355}{5.043}\right) = P(z \geq -2.35) = 1 - 0.0094 = \underline{0.9906}$

(b) $P(x \geq 30) = P\left(z \geq \dfrac{29.5 - 26.355}{5.043}\right) = P(z \geq 0.62) = 1 - 0.7324 = \underline{0.2676}$

(c) $P(25 \leq x \leq 35) = P\left(\dfrac{24.5 - 26.355}{5.043} \leq z \leq \dfrac{35.5 - 26.355}{5.043}\right) = P(-0.37 \leq z \leq 1.81)$
$= 0.9649 - 0.3557 = \underline{0.6092}$

(d) $P(x > 40) = P\left(z > \dfrac{40.5 - 26.355}{5.043}\right) = P(z > 2.81) = 1 - 0.9975 = \underline{0.0025}$

Chapter 7

Sect 7.1

#3 parameter - is a numerical value that describes a population

examples μ, σ^2, σ, ρ (correlation)
+ measures of skewness, kurtosis, etc.

#5 Statistical Inference is a conclusion about the value of a population parameter based upon information about the corresponding sample statistic and its probability distribution. In later chapters there are topics of point and interval estimation and hypothesis testing.

Sect 7.2

#13 $x = 85$ $\sigma = 25$

(a) $P(x < 40) = P(z < \frac{40-85}{25}) = P(z < -1.8) = \underline{0.0359}$

(b) $n = 2$ $P(\bar{x} < 40) = P(z < \frac{40-85}{15.87}) = P(z < -2.55) = \underline{0.0054}$

$\mu_{\bar{x}} = 85$

$\sigma_{\bar{x}} = \frac{25}{\sqrt{2}} = 15.87$

(c) $n = 3$ $\mu_{\bar{x}} = 85$ $\sigma_{\bar{x}} = \frac{25}{\sqrt{3}} = 14.43$ $P(\bar{x} \le 40) = P(z \le -3.12) = \underline{0.0009}$

(d) $n = 5$ $\mu_{\bar{x}} = 85$ $\sigma_{\bar{x}} = \frac{25}{\sqrt{5}} = 11.18$ $P(\bar{x} \le 40) = P(z \le -4.02) \cong \underline{0.0002}$

(e) yes, decreased

#17 $\mu = 1.6\%$ $\sigma = 0.9\%$ monthly percentage return Templeton World

(a) x is a mean of a sample of size $n = 250$

(b) $P(1.0\% < \bar{x} < 2.0\%) = P(\frac{1.0-1.6}{0.367} < z < \frac{2.0-1.6}{0.367}) = P(-1.63 < z < 1.09)$
$= 0.8599 - 0.0516 = \underline{0.8083}$

$\sigma_{\bar{x}} = \frac{0.9}{\sqrt{6}} = 0.367$

(c) $\sigma_{\bar{x}} = \frac{0.9}{\sqrt{24}} = 0.184$ $P(\frac{1-1.6}{0.184} < z < \frac{2.0-1.6}{0.184}) = P(-3.26 < z < 2.17)$
$= 0.9850 - 0.0006 = \underline{0.9844}$

(d) increase - more likely to be near mean of 1.6%

(e) $P(\bar{x} < 1\%) = P(z < -3.26) = \underline{0.0006}$ yes - very unlikely

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Sect 7.3 #9 $p = 0.06$ $n = 100$

(a) $np = 6 > 5$, $n(1-p) = 94 > 5$

$\mu_{\hat{p}} = 0.06$ $\sigma_{\hat{p}} = \sqrt{\frac{0.06(0.94)}{100}} = 0.0237$

(b) $P(0.07 \leq \hat{p}) = P\left(z \geq \frac{0.065 - 0.06}{0.0237}\right) = P(z \geq 0.21) = 1 - .5832 = 0.4168$

(c) $P(0.11 \leq \hat{p}) = P\left(z \geq \frac{0.105 - 0.06}{0.0237}\right) = P(z \geq 1.90) = 1 - 0.9713 = 0.0287$

yes, the probability of this proportion of defectives is only about 3%

Chapter 8

Sect 8.1 #15 $n = 30$ $\bar{x} = 138.5$ $\sigma = 42.6$ (known)

(a) 90% CI $\bar{x} \pm z_{.05} \frac{\sigma}{\sqrt{n}} \Rightarrow 138.5 \pm 1.64 \frac{42.6}{\sqrt{30}} = 138.5 \pm 12.8$

or 125.7 to 151.3

(b) 95% CI use $z_{.025} = 1.96 \Rightarrow 138.5 \pm 1.96 \frac{42.6}{\sqrt{30}} \Rightarrow$ 123.3 to 153.7

(c) 99% CI use $z_{.005} = 2.58 \Rightarrow 138.5 \pm 2.58 \frac{42.6}{\sqrt{30}} \Rightarrow$ 118.4 to 158.6

(d) yes (incr CI level $\Rightarrow$ larger intervals)

(e) yes

Sect 8.2 #13 (a) $\bar{x} = 91.0$ $s \approx 30.7$ $n = 6$ $df = 6 - 1 = 5$

(b) 75% CI $\bar{x} \pm t_{.125,5} \frac{s}{\sqrt{n}} = 91.0 \pm 1.301 \frac{30.7}{\sqrt{6}}$

$= 91.0 \pm 16.3$ or 74.7 to 107.3

Sect 8.3 #5 (a) $\hat{p} = \frac{39}{62}$ extroverts $\hat{p} = 0.6290$

(b) $\hat{p} \pm z_{.025} \sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow 0.6290 \pm 1.96 \sqrt{\frac{0.6290(0.3710)}{62}}$

0.6290 ± 0.1202 or 0.5088 to 0.7492

(c) $n\hat{p} = 39$ $\therefore$ $np > 5$ is likely
$n\hat{q} = \frac{23}{62}$ $\therefore$ $nq > 5$ is likely
} yes

needed for normal approximation to work

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Sect 8.3 #22 * (a) $\tilde{p} = \frac{20+2}{50+4} = \frac{22}{54} = 0.4074$

$\sqrt{\frac{\tilde{p}\tilde{q}}{n+4}} = 0.0669$ $\quad 0.4074 \pm 1.96\,(0.0669)$

or $\quad 0.2768$ to 0.5385

(b) traditional $\hat{p} = \frac{20}{50} = 0.4$ $\quad \sqrt{\frac{0.4(0.6)}{50}} = 0.0693$

$0.4 \pm 1.96\,(0.0693) = 0.2642$ to 0.5358

(c) $\hat{p} = 0.4$ $\tilde{p} = 0.4074 \Rightarrow \tilde{p}$ is closer to $\frac{1}{2}$

margin of error $0.0693 > 0.0669 \Rightarrow$ plus 4 also narrower

Sect 8.4 #7 (a) $\bar{x}_1 = 747.5$ $\quad s_1 = 170.4$ $\quad \bar{x}_2 = 738.9$ $\quad s_2 = 212.1$ $\quad (n_1 = 12, n_2 = 16)$

(b) $\bar{x}_1 - \bar{x}_2 = 747.5 - 738.9 = 8.6$ $\quad E = t_{0.1,11}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

$= 1.796\sqrt{\frac{170.4^2}{12} + \frac{212.1^2}{16}} = 129.90$

8.6 ± 129.90 or -121.3 to 138.5

(c) because $\pm$ of 0 one region is no more interesting than the other

(d) t distribution (σ_1, σ_2 unknown)

#22 $\hat{p}_1 = \frac{270}{474} = 0.5696$, $\hat{p}_2 = \frac{270}{805} = 0.3354$

(a) 95% CI $\quad 0.5696 \pm 1.96\sqrt{\frac{0.5696\,(0.4304)}{474}} = 0.5696 \pm 0.0446$

0.5250 to 0.6142

(b) 95% CI $\quad 0.3354 \pm 1.96\sqrt{\frac{0.3354\,(0.6646)}{805}} = 0.3354 \pm 0.0326$

0.3028 to 0.3680

(c) $1.96\sqrt{\frac{0.5696(0.4304)}{474} + \frac{0.3354(0.6646)}{805}} = 1.96\,(0.0282) = 0.0552$

$\hat{p}_1 - \hat{p}_2 = 0.2342$ $\quad 0.2342 \pm 0.0552$ $\quad 0.1790$ to 0.2894

(d) at the 95% CI it appears that separated nesting boxes (#1) yield more successful hatches than those closely together (#2)