

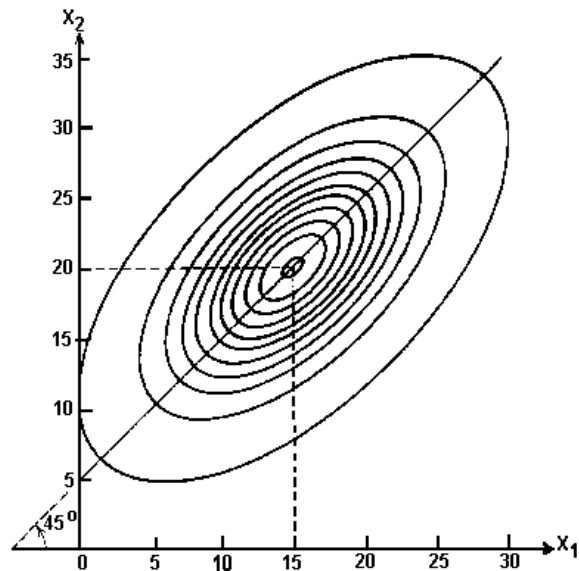
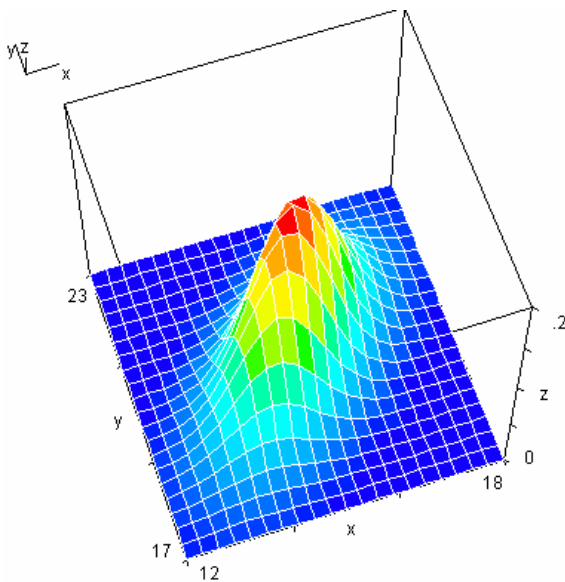
Multivariate Normal Distribution

$$f(x,y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left[-\frac{1}{2(1-\rho^2)}\left\{\frac{(x-\mu_1)^2}{\sigma_1^2} + \frac{(y-\mu_2)^2}{\sigma_2^2} - 2\rho\frac{(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2}\right\}\right]$$

$$\frac{(x-\mu_1)^2}{\sigma_1^2} + \frac{(y-\mu_2)^2}{\sigma_2^2} - 2\rho\frac{(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2} = C \text{ represents an ellipse with center at the point } (\mu_1, \mu_2)$$

which is called the *centroid* of the bivariate population, and major or minor axis along the line passing through this point and making the following angle with the positive x-axis:

$$\theta = \begin{cases} \frac{1}{2} \text{Arc tan } \frac{2\rho\sigma_1\sigma_2}{\sigma_1^2 - \sigma_2^2} & \text{when } \sigma_1 \neq \sigma_2 \\ 45^\circ & \text{when } \sigma_1 = \sigma_2 \end{cases} \text{ . This line contains the major axis if } \rho > 0 \text{ and the minor axis if } \rho < 0.$$



Several members of the family of ellipses:
 $(x - 15)^2 + (y - 20)^2 - 2(0.6)(x - 15)(y - 20) = C$

Properties:

$$\text{Cov}(X, Y) = \rho \quad X, Y \text{ independent if and only if } \rho = 0$$

The conditional distribution of Y given X = x is normal with mean and variance given by:

$$E(Y | X = x) = \mu_2 + \rho\sigma_2\left(\frac{x - \mu_1}{\sigma_1}\right), \quad \text{Var}(Y | X = x) = \sigma_2^2(1 - \rho^2)$$

In general, for n dimensions $f(x_1, x_2, \dots, x_n) = (2\pi)^{-n/2} |\Sigma|^{-1/2} \exp(-\chi^2 / 2)$ where

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \cdots & \rho_{1n}\sigma_1\sigma_n \\ \rho_{21}\sigma_2\sigma_1 & \sigma_2^2 & \cdots & \rho_{2n}\sigma_2\sigma_n \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1}\sigma_n\sigma_1 & \rho_{n2}\sigma_n\sigma_2 & \cdots & \sigma_n^2 \end{bmatrix}, \quad \chi^2 = \mathbf{X}'\Sigma^{-1}\mathbf{X}, \quad \text{and } \mathbf{X} = \begin{bmatrix} x_1 - \mu_1 \\ x_2 - \mu_2 \\ \vdots \\ x_n - \mu_n \end{bmatrix}$$