

## WAVE EQUATION (1 - dimensional) on [0, L]

$$u_{tt} = c^2 u_{xx} \quad \text{where } c^2 = \frac{T}{\rho}$$

given  $T$ =tension and  $\rho = \frac{\text{mass (of undeflected string)}}{\text{unit length}}$

initial displacement:  $u(x, 0) = f(x)$

initial velocity:  $U_t(x,0) = g(x)$  {may be 0}

Solutions:

[1] Fourier Series:

$$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t) = \sum_{n=1}^{\infty} (A_n \cos \lambda_n t + B_n \sin \lambda_n t) \sin\left(\frac{n\pi x}{L}\right)$$

where  $A_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx$

and  $B_n = \frac{2}{cn\pi} \int_0^L g(x) \sin \frac{n\pi x}{L} dx$

[2] D'Alembert's Solution:

$$u(x,t) = \frac{1}{2} [f(x+ct) + f(x-ct)] + \frac{1}{2c} \int_{x-ct}^{x+ct} g(s) ds$$

where  $f(x)$  is the odd function extension with period =  $2L$

[3] Numerical - see attached pages from Kreysig's Advanced Engineering Mathematics (Wiley).