COMPARING TRAPEZOID, SIMPSON, & ROMBERG INTEGRATION

$$\int_0^2 x^4 \ln\left(x + \sqrt{x^2 + 1}\right) dx$$

TRAPEZOID:

n	T_n	$\lvert T_n - T_{n/2} \rvert / 3$
1	23.0982…	
2	12.4305…	3.656…
4	9.2545…	1.058…
8	8.4306…	0.274…
16	8.2228…	0.069…
…	…	…
128	8.154449	1.085×10^{-3}
…	…	…
4096	8.15336519	1.065×10^{-6}

SIMPSON:

n	S_n	$\lvert S_n - S_{n/2} \rvert / 15$
2	8.8746…	
4	8.1959…	0.045…
8	8.1560…	2.660×10^{-3}
16	8.15352516	1.620×10^{-4}
32	8.15337418	1.007×10^{-5}
64	8.15336475	6.283×10^{-7}
128	8.15336417	3.899×10^{-8}

ROMBERG:

$R_{1,1} = T_1 = 23.098167$ $R_{2,2}=8.8745537$ $R_{3,3}=8.1506162$ $R_{4,4}=8.1533369$ $R_{5,5}=8.1533644$
$R_{2,1} = T_2 = 12.430457$ $R_{3,2}=8.1958623$ $R_{4,3}=8.1532944$ $R_{5,4}=8.1533643$
$R_{3,1} = T_4 = 9.2545110$ $R_{4,2}=8.1559549$ $R_{5,3}=8.1533632$
$R_{4,1} = T_8 = 8.4305939$ $R_{5,2}=8.1535252$
$R_{5,1} = T_{16} = 8.2227924$

error estimates:	actual error
$\lvert R_{2,2} - R_{2,1} \rvert = 3.5559033$	0.7211696
$\lvert R_{3,3} - R_{3,2} \rvert = 0.0452461$	0.0027479
$\lvert R_{4,4} - R_{4,3} \rvert = 0.0000425$	0.0000272
$\lvert R_{5,5} - R_{5,4} \rvert = 0.0000001$	0.0000003

Consider: Try $J = \int_0^{\pi} \sin(32x)dx$ on each of the following: Simpson, Romberg, and Adaptive Quadrature.