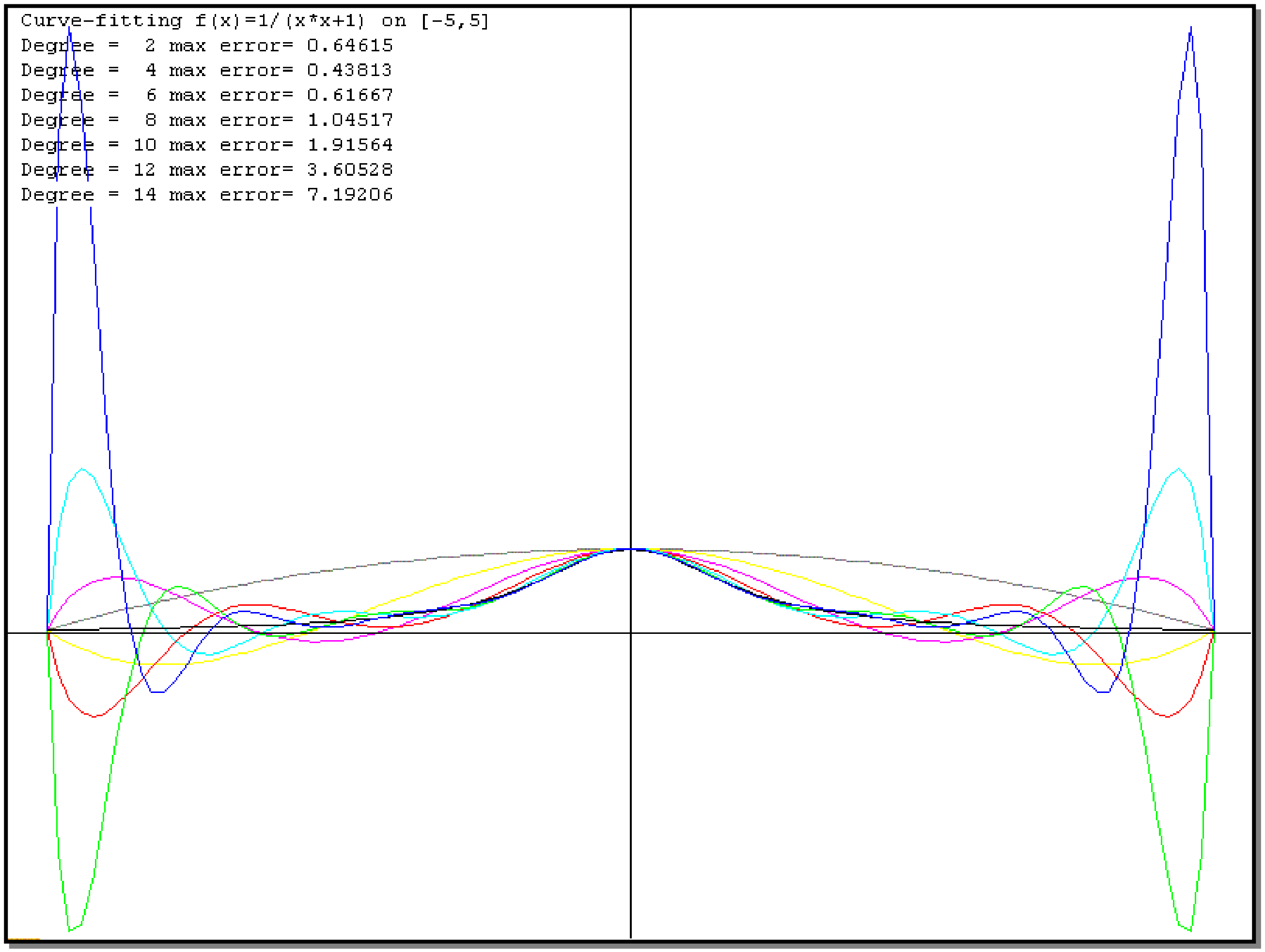
ERRORS IN POLYNOMIAL INTERPOLATION – **Prof. R. Goldstein**

- If the function f is well behaved, can we assume that $\lim_{n\to\infty} \max_{x\in[a,b]} |f(x) - p_n(x)| = 0$. That is, if a function f(x) is fit on an interval [a, b] with more and more nodes does the fit always get better?

Consider: Runge function $f(x) = \dfrac{1}{x^2+1}$ on [-5, 5]

Case #1: Equally spaced points

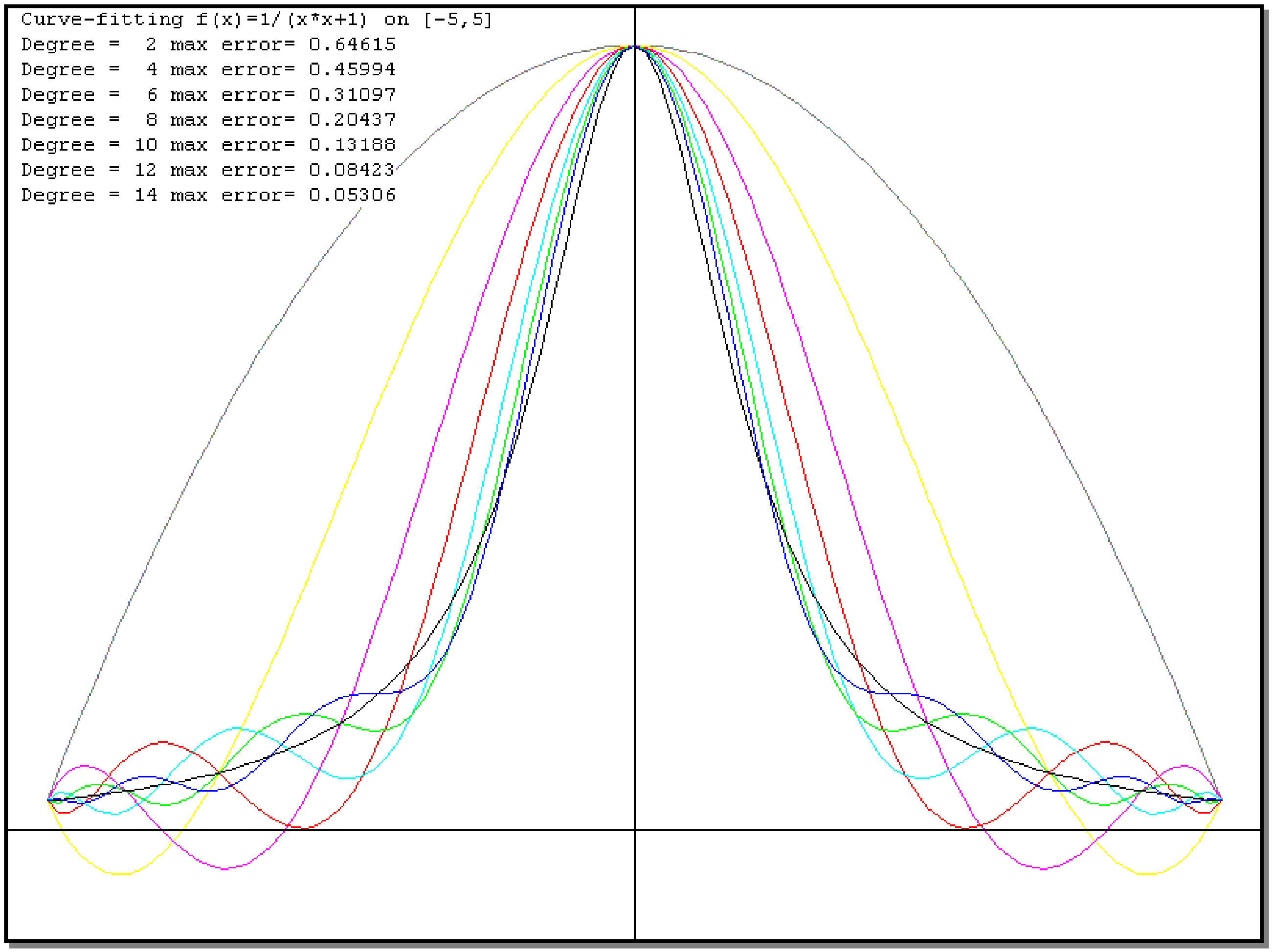


For degree 20 the error grows to 58.3, by 25 it is 75.8, and by 28 it is over 1000. In fact, in this case $\lim_{n\to\infty} \max_{x\in[a,b]} |f(x) - p_n(x)| = \infty$.

Case #2: Unequally spaced points chosen by

$$x_i = -5 * \cos(\tfrac{i\pi}{n}),\ 0 \le i \le n$$

which are called **Chebychev nodes**



Now it does appear that

$$\lim_{n\to\infty} \max_{x\in[a,b]} |f(x) - p_n(x)| = 0.$$