GAUSS QUADRATURE - Prof. Richard B. Goldstein

Formula for [-1,1]

$$\int_{-1}^{1} f(x)\,dx = \sum_{j=1}^{n} w_j f(x_j) + R_n \text{ where } x_j = j^{th} \text{ root of } P_n(x) \text{ and } w_j = \frac{2}{(1-x_j^2)\left[P_n'(x_j)\right]^2}$$

$$\text{and } R_n = \frac{2^{2n+1}(n!)^4}{(2n+1)\left[(2n)!\right]^3} f^{(2n)}(c) \quad \text{(See text for a table of x's and w's)}$$

Formula for [a,b]

$$\int_{a}^{b} f(y)\,dy = \frac{b-a}{2}\sum_{j=1}^{n} w_j f(y_j) + R_n \quad \text{where } y_j = \left(\frac{b-a}{2}\right)x_j + \left(\frac{b+a}{2}\right)$$

GAUSS-LAGUERRE QUADRATURE

$$\int_{-\infty}^{\infty} e^{-x} f(x)\,dx = \sum_{j=1}^{n} w_j f(x_j) + R_n \text{ where } x_j = j^{th} \text{ root of } L_n(x) \text{ and } R_n = \frac{(n!)^2}{(2n)!} f^{(2n)}(c)$$

n	x_j	w_j
2	0.58578 64376	0.85355 33906
	3.41421 35624	0.14544 66094
3	0.41577 45568	0.71109 30099
	2.29428 03603	0.27851 77336
	6.28994 50829	0.01038 92560
4	0.32254 76896	0.60315 41043
	1.74576 11012	0.35741 86924
	4.53662 02969	0.03888 79085
	9.39507 09123	0.00053 92947

EXAMPLES

[1] $$\int_1^4 \sqrt{x}\,dx = \frac{14}{3} = 4.666\ldots$$

For 2 points:

$$\frac{b-a}{2} = \frac{4-1}{2} = 1.5, \quad \frac{b+a}{2} = \frac{4+1}{2} = 2.5$$

$$J_2 \approx 1.5\left\{(1)f(1.5x_1 + 2.5) + (1)f(1.5x_2 + 2.5)\right\}$$

$$\approx 1.5\left\{f(1.5(-.577\ldots) + 2.5) + f(1.5(.577\ldots) + 2.5)\right\}$$

$$\approx 1.5\left\{\sqrt{1.633974596} + \sqrt{3.366025404}\right\}$$

$$\approx 4.669\ 418\ 93$$

Similarly,

$$J_3 \approx 4.666\ 829\ 05$$

$$J_4 \approx 4.666\ 678\ 26$$

$$J_5 \approx 4.666\ 667\ 58$$

[2] $$\int_0^\infty \sqrt{x}\,e^{-x}\,dx = \frac{\sqrt{\pi}}{2} \approx 0.866\ 226\ 93$$

$$J_2 \approx 0.85355\ldots\sqrt{0.58578\ldots} + 0.14544\ldots\sqrt{3.41421\ldots}$$

$$\approx 0.922\ 031\ 77$$

$$J_3 \approx 0.906\ 440\ 45$$

$$J_4 \approx 0.899\ 280\ 22$$