NUMERICAL - FIRST ORDER ODE - Prof. Richard B. Goldstein

PROBLEM: Approximate y(t) on a # t # b for $\frac{dy}{dt} = f(t, y)$, given $y(a) = w_0$

SOLUTIONS: Let t_i = a + ih for i = 0, 1, 2, ..., N where h = (b - a)/N
Let w_i . $y(t_i)$

[1] EULER'S METHOD Errors of $\mathcal{O}(h)$

$$w_{i+1} = w_i + hf(t_i, w_i)$$

[2] CORRECTED EULER / HEUN'S METHOD Errors of $\mathcal{O}(h^2)$

$$k_1 = hf(t_i, w_i)$$
$$k_2 = hf(t_i + h, w_i + k_1)$$
$$w_{i+1} = w_i + \frac{1}{2}(k_1 + k_2)$$

[3] RUNGE-KUTTA (Fourth Order) Errors of $\mathcal{O}(h^4)$

$$k_1 = hf(t_i, w_i)$$
$$k_2 = hf\left(t_i + \frac{h}{2}, w_i + \frac{k_1}{2}\right)$$
$$k_3 = hf\left(t_i + \frac{h}{2}, w_i + \frac{k_2}{2}\right)$$
$$k_4 = hf(t_i + h, w_i + k_3)$$
$$w_{i+1} = w_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

[4] Taylor Series (Arbitrary Order)

$$y(t_i + h) = y(t_i) + hy'(t_i) + \frac{h^2}{2!}y''(t_i) + \frac{h^3}{3!}y'''(t_i) + \ldots + \frac{h^n}{n!}y^{(n)}(t_i) + E_n$$
$$w_{i+1} = w_i + hf(t_i, w_i) + \frac{h^2}{2}\left.\frac{df}{dt}\right|_{(t_i, w_i)} + \frac{h^3}{6}\left.\frac{d^2f}{dt^2}\right|_{(t_i, w_i)} + \ldots + \frac{h^n}{n!}\left.\frac{d^{n-1}f}{dt^{n-1}}\right|_{(t_i, w_i)}$$

where the derivatives are found by **implicit differentiation**

- Higher Order and systems of ODE can be solved by R-K
- Partial Differential Eqs. are also solved by difference eqs. - categorized as hyperbolic, elliptic & parabolic - these sometimes require large matrices