MATH/CS 450 – PROF. RICHARD B. GOLDSTEIN – CHAP 4 HW – Burden + Faires 8^{th} Ed

SECT 4.1

#5b

x	$f(x)$	$f'(x)$
8.1	16.94410	3.09205
8.3	17.56492	3.11615
8.5	18.19056	3.13998
8.7	18.82091	3.16357

$$f'(8.1) \approx \frac{1}{0.2}\left[-\tfrac{3}{2}f(8.1) + 2f(8.3) - \tfrac{1}{2}f(8.5)\right]$$

$$f'(8.3) \approx \frac{1}{0.2}\left[-\tfrac{1}{2}f(8.1) + \tfrac{1}{2}f(8.5)\right]$$

$$f'(8.5) \approx \frac{1}{0.2}\left[-\tfrac{1}{2}f(8.3) + \tfrac{1}{2}f(8.7)\right]$$

$$f'(8.7) \approx \frac{1}{0.2}\left[\tfrac{1}{2}f(8.3) - 2f(8.5) + \tfrac{3}{2}f(8.7)\right]$$

#7b $f(x) = x\ln x$

$f'(x) = x \cdot \frac{1}{x} + 1 \cdot \ln x = 1 + \ln x$ for 8.1 $1 + \ln 8.1 = 3.09186$ etc.

x	actual error	error bound
8.1	0.00019	0.00020
8.3	0.00011	0.00011
8.5	0.00009	0.00010
8.7	0.00021	0.00020

Error bound at endpts. $\frac{h^2}{3}f^3(c)$

at interior pts $\frac{h^2}{6}f^3(c)$

$f''(x) = x^{-1}$ $f'''(x) = -x^{-2}$ ← abs max occurs at left pt.

error bdd for 8.1 $\frac{(0.2)^2}{3}\left(\frac{1}{8.1}\right)^2 = 0.00020$

for 8.3 $\frac{(0.2)^2}{6}\left(\frac{1}{8.1}\right)^2 = 0.00011$

for 8.5 $\frac{(0.2)^2}{6}\left(\frac{1}{8.3}\right)^2 = 0.00010$

+ for 8.7 $\frac{(0.2)^2}{3}\left(\frac{1}{8.3}\right)^2 = 0.00020$

#18

x	$f(x)$
0.2	0.979 865 2
0.4	0.917 771 0
0.6	0.808 034 8
0.8	0.638 609 3
1.0	0.384 373 5

$$f'(0.4) \approx \frac{f(0.6) - f(0.2)}{0.4} \approx \boxed{-0.429576}$$

$$f''(0.4) \approx \frac{f(0.6) - 2f(0.4) + f(0.2)}{(0.2)^2} \approx \boxed{-1.19105}$$

$$f'(0.6) \approx \frac{f(0.8) - f(0.4)}{2(0.2)} \approx \boxed{-0.6979043}$$

$$f'(0.6) = [5\text{ pt.}]\ \frac{1}{12(0.2)}\left[-f(1.0) + 8f(0.8) - 8f(0.4) + f(0.2)\right] = \boxed{-0.68241746}$$

$$f''(0.6) \approx \frac{f(0.8) - 2f(0.6) + f(0.4)}{(0.2)^2} = \boxed{-1.4922325}$$

Sect 4.2

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#1c $f(x) = 2^x \sin x \quad x_0 = 1.05 \quad h = 0.4$

$N_1(0.4) = N_1(h) \simeq \frac{f(1.45) - f(0.65)}{0.8} = \frac{2.712\ 171\ 818 - (+0.94963926)}{0.8} = 2.203\ 165\ 697$

$N_1(0.2) = N_1\left(\frac{h}{2}\right) = \frac{f(1.25) - f(0.85)}{0.4} = \frac{2.257\ 078\ 523 - 1.35\ 4\ 183\ 625}{0.4} = 2.257\ 237\ 243$

$N_1(0.1) = N_1\left(\frac{h}{4}\right) = \frac{f(1.15) - f(0.95)}{0.2} = \frac{2.025550007 - 1.571415173}{0.2} = 2.270\ 674\ 167$

$N_2(0.4) \approx \frac{4N_1(0.2) - N_1(0.4)}{3} = 2.275\ 261\ 092$

$N_2(0.2) = \frac{4N_1(0.1) - N_1(0.2)}{3} = 2.275\ 153\ 142$

$N_3(0.4) = \frac{16N_2(0.2) - N_2(0.4)}{15} = \boxed{2.275\ 145\ 945}$

#2c

$N_1(0.05) = N_1\left(\frac{h}{8}\right) = \frac{f(1.1) - f(1)}{0.1} = \frac{1.910\ 344\ 796 - 1.682\ 941\ 97}{0.1} = \boxed{2.274\ 028\ 266}$

$N_2(0.1) = \frac{4N_1(0.05) - N_1(0.1)}{3} = 2.275\ 146\ 300$

$N_3(0.2) = \frac{16N_2(0.1) - N_2(0.2)}{15} = 2.275\ 145\ 843$

$N_4(0.4) = \frac{64N_3(0.2) - N_3(0.4)}{63} = \boxed{2.275\ 145\ 842}$

note $f'(x) = 2^x \cos x + 2^x \sin x \cdot (\ln 2)$

$f'(1.05) = 2^{1.05}\left[\cos(1.05) + (\ln 2)\sin(1.05)\right] = \underline{\underline{2.275\ 145\ 842}}$

actual answer by calculus

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Sect 4.3

#1a $\displaystyle\int_1^{1.5} x^2 \ln x\,dx \simeq \frac{0.5}{2}\left(1^2 \ln 1 + 1.5^2 \ln 1.5\right) = \underline{0.228\,074\,123}$

#3a $f = x^2 \ln x$ $\quad f'(x) = 2x\ln x + x^2 \cdot \frac{1}{x} = 2x\ln x + x$

$f''(x) = 2\ln x + 2x \cdot \frac{1}{x} + 1 = \underline{2\ln x + 3}$

max at $x = 1.5$ $\quad f''(1.5) = 3.811$

$\dfrac{(0.5)^3}{12}(3.811) = \underline{0.0397}$

#5c $\displaystyle\int_1^{1.5} x^2 \ln x\,dx \approx \frac{0.25}{3}\left[f(1) + 4f(1.25) + f(1.5)\right] = \underline{0.192\,245\,307}$

$h = \frac{1.5-1}{2} = 0.25$ $\quad f'''(x) = 2x^{-1}$ $\quad f^{IV}(x) = -2x^{-2}$

max at $x = 1.0$ is 2.0

$E = \dfrac{(0.25)^5}{90}(2.0) = \underline{0.000\,007\,111}$

#9c $\displaystyle\int_1^{1.5} x^2 \ln x\,dx \simeq 0.5\,(1.25)^2 \ln 1.25 = \underline{0.174\,330\,899}$ $\quad h = 0.25$

$E = \dfrac{(0.25)^3}{3}(3.811) = \underline{0.0198}$

Sect 4.4

#1f $\displaystyle\int_1^3 \frac{x}{x^2+4}\,dx$ $\quad n = 8$ $\quad h = \frac{3-1}{8} = 0.25$ $\quad f(x) = \frac{x}{x^2+4}$

$\frac{0.25}{2}\left[f(1) + 2f(1.25) + 2f(1.5) + 2f(1.75) + 2f(2) + 2f(2.25) + 2f(2.5) + 2f(2.75) + f(3)\right]$

$= \frac{0.25}{2}\left[0.2 + 2(0.224719101) + \dots + 0.230769231\right]$

$= \frac{0.25}{2}\left[3.815\,814\,932\right] = \boxed{0.476\,976\,866}$

actual $\left.\dfrac{\ln(x^2+4)}{2}\right|_1^3 = \dfrac{\ln 13 - \ln 5}{2}$
$= 0.477\,755\,722$

#3f $\frac{0.25}{3}\left[f(1) + 4f(1.25) + 2f(1.5) + 4f(1.75) + 2f(2) + 4f(2.25) + 2f(2.5) + 4f(2.75) + f(3)\right]$

$= \frac{0.25}{3}\left[5.733\,055\,755\right] = \boxed{0.477\,754\,646}$

Sect 4.4 continued

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#5f $n=8$ $n+2=10$ $h=\frac{3-1}{8+2}=0.2$

$x_0=1.2$ $x_1=1.6$ $x_2=2.0$ $x_3=2.4$ $x_4=2.8$

$=2(0.2)\left[f(1.2)+f(1.6)+f(2)+f(2.4)+f(2.8)\right]$

$=2(0.2)\left[0.220588235+\ldots\right] = 0.4\,(1.1968788) = \boxed{0.47875152}$

Sect 4.5 #1a

$R_{3,3}$ for $\int_1^{1.5} x^2 \ln x \, dx$

$R_{1,1} = T_1 = \frac{0.5}{2}\left(f(1)+f(1.5)\right) = 0.228\ 074\ 123$ (same as 4.3 #1c)

$T_2 = R_{2,1} = \frac{1}{2}\left[R_{1,1} + 0.5 f(1.25)\right] = 0.201\ 202\ 511$

$T_4 = R_{3,1} = \frac{1}{2}\left[R_{2,1} + 0.25\left\{f(1.125)+f(1.375)\right\}\right] = 0.194\ 494\ 473$

$R_{2,2} = \frac{4R_{2,1}-R_{1,1}}{3} = 0.192\ 245\ 307$

$R_{3,2} = \frac{4R_{3,1}-R_{2,1}}{3} = 0.192\ 258\ 460$

$R_{3,3} = \frac{16R_{3,2}-R_{2,2}}{15} = \boxed{0.192\ 259\ 337}$

Actual value is
$\left(\frac{x^3}{3}\ln x - \frac{x^3}{9}\right)\Big|_1^{1.5}$
$= 0.192\ 259\ 357$

from int. by parts
$u=\ln x$ $dv = x^2 \ln x$

Sect 4.6

#1a $S(1,1.5) = \frac{0.25}{3}\left[f(1)+4f(1.25)+f(1.5)\right] = 0.192\ 245\ 31$

$S(1,1.25) = \frac{0.125}{3}\left[f(1)+4(1.125)+f(1.25)\right] = 0.039\ 372\ 43$

$S(1.25,1.5) = \frac{0.125}{3}\left[f(1.25)+4f(1.375)+f(1.5)\right] = 0.152\ 886\ 03$

$\Big\}\ 0.192\ 258\ 46$

error est $\approx \frac{1}{15}\left|.192\ 258\ 46 - .192\ 245\ 31\right| = 8.767\times10^{-7}$ > close

actual error $= \left|0.192\ 259\ 36 - 0.192\ 258\ 46\right| = 9\times10^{-7}$

#2a — already at accuracy

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SECT 4.7

#1a, $n=2$

$$\int_1^{1.5} x^2 \ln x \, dx = \int_{-1}^{1} f\left[\frac{(1.5-1)t + 1.5+1}{2}\right]\left(\frac{1.5-1}{2}\right) dt \qquad \left(\frac{b-a}{2} = 0.25,\ \frac{b+a}{2} = 1.25\right)$$

$= 0.25t + 1.25$

$$\cong 0.25\left[1 \cdot f(0.25 \cdot 0.577350269 + 1.25) + 1 \cdot f(0.25 \cdot -0.577\ldots + 1.25)\right]$$

$$\approx 0.25\left[f(1.3943376) + f(1.1056624)\right] = \boxed{0.1922687}$$

#2a, $n=3$

$$I \approx 0.25\left[\tfrac{5}{9} f(1.4436492) + \tfrac{8}{9} f(1.25) + \tfrac{5}{9} f(1.0563508)\right] = \boxed{0.1922594}$$

#3a, $n=4$

$$I \approx 0.25\left[0.34785485\{f(1.4652841) + f(1.0347159)\} + 0.65214515\{f(1.3349953) + f(1.1650047)\}\right]$$

$$= 0.25\left[0.34785485(0.856819) + 0.65214515(0.7222147)\right] = \boxed{0.1922594}$$

= exact

SECT 4.8

#1c

$x \backslash y$	y_1	y_2	y_3	y_4	y_5
2.00	12.0	19.625	31	46.875	68
2.05	12.817625	21.028916	33.278547	50.374186	73.1235
2.10	13.671	22.497891	35.665875	54.093172	78.498
2.15	14.560875	24.033789	38.164516	57.885979	84.1295
2.20	15.488	25.636875	40.777	61.90668	90.024

for $x = 2.0$:
$x = y_1 = 2.0$
$y_2 = 2.5$
$y_3 = 3.0$
$y_4 = 3.5$
$2x = y_5 = 4.0$

for $x = 2.05$:
$y_1 = 2.0500$
$y_2 = 2.5625$
$y_3 = 3.0750$
$y_4 = 3.5875$
$y_5 = 4.1000$
etc

$$\int_2^{2.2}\int_x^{2x} x^2 + y^3 \, dx \approx \qquad \left(\underline{\text{exact}}\ \left(\frac{x^4}{4} + \frac{3x^5}{4}\right)\Big|_2^{2.2} = 16.50864\right)$$

$$\begin{array}{ccccccccc}
1 & - & 4 & - & 2 & - & 4 & - & 1 \\
4 & - & 16 & - & 8 & - & 16 & - & 4 \\
2 & - & 8 & - & 2 & - & 8 & - & 2 \\
4 & - & 16 & - & 8 & - & 16 & - & 4 \\
1 & - & 4 & - & 2 & - & 4 & - & 1
\end{array}$$

$$\frac{0.05}{3}\Big\{\frac{0.5}{3}\left[(1)12 + (4)19.625 + (2)31 + (4)46.875 + (1)68\right]$$

$$+ \frac{0.5125}{3}\left[(4)12.817625 + (16)21.028916 + (8)33.278547 + (16)50.374186 + (4)73.1235\right]$$

$$+ \frac{0.525}{3}\left[(2)13.671 + (8)22.497891 + (2)35.665875 + (8)54.043172 + (2)78.498\right]$$

$$+ \frac{0.5375}{3}\left[(4)14.560875 + (16)24.033789 + (8)38.164516 + (16)57.885979 + (4)84.1295\right]$$

$$+ \frac{0.55}{3}\left[(1)15.488 + (4)25.636875 + (2)40.777 + (4)61.906625 + (1)90.024\right]\Big\}$$

$$= \frac{0.05}{3}\Big\{\frac{0.5}{3}[408] + \frac{0.5125}{3}[1752.4425] + \frac{0.525}{3}[939.332] + \frac{0.5375}{3}[2010.7875] + \frac{0.55}{3}[537.24]\Big\} = \boxed{16.5086406}$$

Sect 4.9 (Opt.)

Mathics 440
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#1 a $\int_0^1 x^{-1/4} \sin x \, dx = \int_0^1 \frac{\sin x}{x^{1/4}} \, dx$ $n=4$

$g(x) = \sin x$

$(x-a)^p = (x-0)^{1/4}$ $\therefore$ $a=0$ $p=0.25$

$P_4(x) = x - \frac{x^3}{6}$ (beginning of Taylor series)

$$\int_0^1 \frac{P_4(x)}{x^{1/4}} dx = \int_0^1 \frac{x - \frac{x^3}{6}}{x^{0.25}} dx = \int_0^1 x^{3/4} - \frac{1}{6} x^{11/4} dx$$

$$= \left(\frac{4}{7} x^{7/4} - \frac{2}{45} x^{15/4}\right)\Big|_0^1 = \frac{4}{7} - \frac{2}{45} = \underline{0.526984127}$$

let $G(x) = \dfrac{\sin x - x + x^3/6}{x^{1/4}}$

<u>n=4</u>

x	$G(x)$
0	0
0,25	0,000 011 491
0,50	0,000 307 852
0,75	0,002 096 765
1	0,008 137 651

$\int_0^1 G(x) dx \approx$

$\frac{0{,}25}{3}\left[G(0) + 4G(0{,}25) + 2G(0{,}5) + 4G(0{,}75) + G(1)\right]$

$= 0.001432198$

$$\int_0^1 \frac{\sin x}{x^{1/4}} dx \simeq 0{,}526\ 984\ 127 + 0{,}001\ 432\ 198 = \boxed{0.528\ 416\ 325}$$