MCS 440 – Prof. Richard B. Goldstein – Burden & Faires – 8th Ed – Chapter 2

Sect 2.1

#3c $f(x) = x^3 - 7x^2 + 14x - 6 = 0$ interval $[3.2, 4]$ tol: 10^{-2}

$f(3.2) = -0.112$
$f(4) = +2.000$ } ∴ keep left negative, right positive

n	a_n (−)	b_n (+)	p_n	$f(p_n)$	$\frac{(b_n - a_n)}{2}$	
1	3.200	4.000	3.600	+0.336 > 0	0.400	(new b)
2	3.200	3.600	3.400	−0.016 < 0	0.200	(new a)
3	3.400	3.600	3.500	+0.125 > 0	0.100	(new b)
4	3.400	3.500	3.450	+0.046 > 0	0.050	(new b)
5	3.400	3.450	3.425	+0.013 > 0	0.025	(new b)
6	3.400	3.425	3.413	−0.001 < 0	0.013	(new a)
7	3.413	3.425	3.419		$0.006 < 10^{-2}$	∴ stop

$f(p_7)$ not needed   root $\approx$ 3.42

#14 $f(x) = x^3 + x - 4 = 0$ interval $[1, 4]$ tol: 10^{-3}

How large must n be? $|p_n - p| = \frac{b-a}{2^n} = \frac{4-1}{2^n} = \frac{3}{2^n} < 10^{-3}$

or $2^n > \frac{3}{10^{-3}} = 3000 \Rightarrow n > \frac{\ln 3000}{\ln 2} = 11.55\ldots$

$n \ln 2 > \ln 3000$ ∴ use 12

$f(1) = -2 < 0$ $f(4) = 64 > 0$ ∴ left (−) + right (+)

n	a_n	b_n	p_n	$f(p_n)$	$(b_n - a_n)/2$
1	1.000	4.000	2.500	+14.125 > 0	1.500
2	1.000	2.500	1.750	+3.110 > 0	0.750
• • •					
10	1.375	1.381	1.378	−0.006 < 0	0.0029
11	1.378	1.381	1.379	+0.004 > 0	0.0015
12	1.378	1.379			$0.0007 < 10^{-3}$

root $\approx$ 1.379

Sect 2.2

#1c $f(x) = x^4 + 2x^2 - x - 3 \Rightarrow g_3(x) = \left(\dfrac{x+3}{x^2+2}\right)^{1/2}$

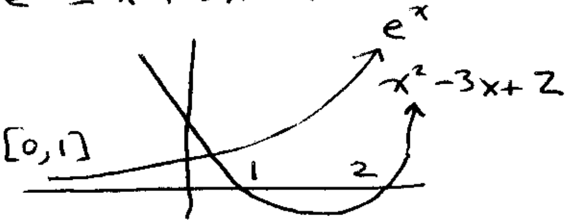
$$x = \left(\frac{x+3}{x^2+2}\right)^{1/2} \Rightarrow x^2 = \frac{x+3}{x^2+2} \Rightarrow x^4 + 2x^2 = x+3$$

$$\Rightarrow x^2 + 2x^2 - x - 3 = 0$$

#11a $x = \dfrac{2 - e^x + x^2}{3}$ is equivalent to $f(x) = e^x - x^2 + 3x - 2 = 0$

let $[0,1] = [a,b]$

graphing gives a root in $[0,1]$



$g(x) = \dfrac{2-e^x+x^2}{3}$ is continuous on $[0,1]$

$$\left.\begin{array}{l} g(0) = 0.333\ldots \in [0,1] \\ g(1) = 0.0939\ldots \in [0,1] \\ g'(x) = \dfrac{-e^x+2x}{3} = 0 \text{ nowhere in } [0,1] \therefore \text{ no local max or min} \end{array}\right\} \Rightarrow \underline{g(x) \in C[0,1]}$$

$$\left.\begin{array}{l} |g'(0)| = \left|-\frac{1}{3}\right| = |-0.333| < 1 \Leftarrow \text{largest absolute value} \\ |g'(1)| = |-0.2394\ldots| < 1 \\ g''(x) = \dfrac{-e^x+2}{3} = 0 \text{ only at } x = \ln 2 = 0.693\ldots \\ |g'(\ln 2)| = |-0.20456\ldots| < 1 \end{array}\right\} |g'(x)| \le \frac{1}{3} < 1$$

by theorem $\exists 1$ f.p (there exists a unique fixed pt.)

use $p_0 = 0.5$

Corollary 2.4 $|p_n - p| \le \left(\frac{1}{3}\right)^n \max(p_0 - 0, 1 - p_0) = \frac{1}{2}\left(\frac{1}{3}\right)^n$

$$\frac{1}{2}\left(\frac{1}{3}\right)^n < 10^{-5} \Rightarrow \left(\frac{1}{3}\right)^n < 2\times 10^{-5} \Rightarrow n\ln(1/3) > \ln(2\times10^{-5}) = -10.82$$

$$\Rightarrow n > 9.85 \Rightarrow \underline{\underline{n = 10}}$$

at most

Starting with $p_0 = 0.5$

$p_1 = 0.2004262$

$p_2 = 0.2727491$

$p_3 = 0.2536072$

$\vdots$

$p_7 = 0.2575123$

$p_8 = 0.2575350$ within 10^{-5}

$p_9 = 0.2575291$

$\Rightarrow$ root $\approx \underline{\underline{0.25753}}$

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Sect 2.3

#5d $f(x) = x - 0.8 - 0.2 \sin x = 0$ in $[0, \frac{\pi}{2}]$

$f'(x) = 1 - 0.2 \cos x$ Use $p_0 = \frac{\pi}{4} = 0.785398$

$$p_{n+1} = p_n - \frac{p_n - 0.8 - 0.2 \sin p_n}{1 - 0.2 \cos p_n}$$

$$p_1 = 0.7853982 - \frac{(-0.156023)}{0.8585786} = 0.9671208$$

Similarly

$p_2 = 0.9643346$

$p_3 = 0.9643339 = p_4$ etc.

#6d $f(x) = (x-2)^2 - \ln x$ $\underline{1 \le x \le 2}$ + $\underline{e \le x \le 4}$

$f'(x) = 2(x-2) - \frac{1}{x}$

Using the midpt. $p_0 = \frac{1+2}{2} = 1.5 \Rightarrow p_1 = 1.5 - \frac{0.155465}{-1.666667} = 1.406721$

similarly $p_2 = 1.412370$, $p_3 = 1.412391$, $p_4 = \underline{1.412391}$ ... (stop)

next use $p_0 = \frac{e+4}{2} = 3.359141$

then $p_1 = 3.359141 - \frac{0.6355788}{2.420587} = 3.096569$

and continuing $p_2 = 3.057980$, $p_3 = \underline{3.057104} = p_4$ (stop)

#7d $\underline{\underline{\text{Secant}}}$ $$p_{n+1} = p_n - \frac{f(p_n)}{\frac{f(p_n) - f(p_{n-1})}{p_n - p_{n-1}}} = p_n - \frac{f(p_n)\,(p_n - p_{n-1})}{f(p_n) - f(p_{n-1})}$$

Using 2 starting pts. at ends $p_0 = 0$ $p_1 = \frac{\pi}{2} = 1.5707963$

$f(0) = -0.8$ $f(\frac{\pi}{2}) = 0.5707963$

$$p_2 = 1.570796 - \frac{0.5707963\,(1.570796 - 0)}{0.5707963 - (-0.8)} = 0.9167025$$

and continuing

$p_3 = 0.9615513$

$p_4 = 0.9643461$

$p_5 = 0.9643339$

$p_6 = 0.9643339$

... $\underline{\text{stop}}$

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Sect 2.3

#9d false position $p_n = b - \frac{f(b)(b-a)}{f(b)-f(a)}$

a	b	f(a)	f(b)	p	f(p)
0	$\frac{\pi}{2} = 1.570796$	-0.8	0.5707963	0.9167205	-0.042002
0.9167205	"	-0.042002	"	0.9615513	-0.002465
0.9615513	"	-0.002465	"	0.9641708	-0.0001445
0.9641708	"	-0.0001445	"	0.9643243	-8.47×10^{-6}
0.9643243	"	-8.47×10^{-6}	"	0.9643333	stop

Sect 2.4

#1a $f(x) = x^2 - 2xe^{-x} + e^{-2x} = (x - e^{-x})^2 \quad 0 \le x \le 1$

$f'(x) = 2x - 2e^{-x} + 2xe^{-x} - 2e^{-2x} = 2x + (2x-2)e^{-x} - 2e^{-2x}$

$f''(x) = 2 + 2e^{-x} + 2e^{-x} - 2xe^{-x} + 4e^{-2x} = 2 + (4-2x)e^{-x} + 4e^{-2x}$

$p_0 = 0.5$

Standard N-R $\quad p_1 = 0.5 - \frac{0.0113488}{-0.3422895} = 0.5331556$

$p_2 = 0.5331556 - \frac{0.0028724}{-0.1700837} = 0.5500437$ (note: goes to zero too)

continuing ... $p_{13} = 0.567135$ (too many iterations)

#3a modified N-R $\quad p_{n+1} = g(p_n) = p_n - \frac{f(p_n)\, f'(p_n)}{[f'(p_n)]^2 - f(p_n)\, f''(p_n)}$

$$p_1 = 0.5 - \frac{(0.01134878)(-0.3422895)}{(-0.3422895)^2 - (0.01134878)(5.2911057)} = 0.56801373$$

$$p_2 = 0.56801373 - \frac{(1.8602\times10^{-6})(0.00427349)}{(0.00427349)^2 - (1.8602\times10^{-6})(4.9072377)}$$

$p_2 = 0.56714343$

$p_3 = 0.5614329 = p_4$ stop

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Sect 2.5

1b $\quad p_n = \left(\dfrac{e^{p_{n-1}}}{3} \right)^{1/2} \qquad p_0 = 0.75$

Aiken's Δ^2 method $\quad \hat{p}_n = p_n - \dfrac{(p_{n+1} - p_n)^2}{p_{n+2} - 2p_{n+1} + p_n}$

n	p_n	$\hat{p}_n$
1	0.8400397	0.909567
2	0.8787224	0.909917
3	0.8958834	0.909989
4	0.9036037	0.910004
5	0.9070984	0.910007
6	0.9086849	
7	0.9094059	

$$\hat{p}_1 = 0.8400397 - \frac{(0.8787224 - 0.8400397)^2}{0.8958834 - 2(0.8787224) + 0.8400397}$$

$$\hat{p}_1 = 0.8400397 - 0.0695276$$

$$\hat{p}_1 = 0.9095673$$

#11 a $\quad x = \dfrac{2 - e^x + x^2}{3} \qquad [0,1]$

$$p_0 = 0.5$$

$$p_1 = \frac{2 - e^{0.5} + (0.5)^2}{3} = 0.2004262$$

$$p_2 = g(p_1) = 0.272749$$

$$p_0' = p_0 - \frac{(p_1 - p_0)^2}{p_2 - 2p_1 + p_0} = \underline{0.2586844}$$

$$\left.\begin{array}{l} p_1' = g(p_0') = 0.2572309 \\ p_2' = g(p_1') = 0.2576080 \end{array}\right\} \Rightarrow p_0''$$

$$p_0'' = p_0' - \frac{(p_1' - p_0')^2}{p_2' - 2p_1' + p_0'} = 0.2586844 - 0.0011541 = \underline{0.2575303}$$

$$p_1'' = g(p_0'') = \underline{0.2575303} \quad \text{(no change } \therefore \text{ stop)}$$

Sect 2.6 $\quad p(x) = x^4 + 2x^2 - x - 3 \qquad p'(x) = 4x^3 + 4x - 1$

start with N-R $\quad \left.\begin{array}{l} p(0) = -3 \\ p(1) = -1 \\ p(2) = +19 \end{array}\right\} \Rightarrow$ root near 1 (I used 1.05)

$$p_0 = 1.05 \Rightarrow \begin{array}{l} p_1 = 1.13039 \\ p_2 = 1.124164 \\ p_3 = 1.124123 \end{array}$$

#1d Continuing using Horner's Method

1.124123	1	0	2	-1	-3
	⇓ +	1.124123	1.2636526	3.668747	+3
	1	1.124123	3.2636526	2.668747	0

no remainder

next consider

$$P(x) = x^3 + 1.124123\,x^2 + 3.2636526\,x + 2.668747 = 0$$

Since $P(0) = 2.66$ and $P(-1) = -0.4707$ } try $p_0 = -0.8$

now $P'(x) = 3x^2 + 2.248246\,x + 3.2636526$

and using N-R

$p_1 = -0.8783631$

$p_2 = -0.8760553$

$p_3 = -0.8760531 = p_4$ stop

-0.8760531	1	1.124123	3.263653	2.668747
		-0.876053	-0.2173224	-2.668747
	1	0.248070	3.046330	≈ 0

$x^2 + 0.248070\,x + 3.046330$ is a quadratic with complex roots $(b^2 - 4ac < 0)$

$$x = -0.124035 \pm 1.740961\, i$$