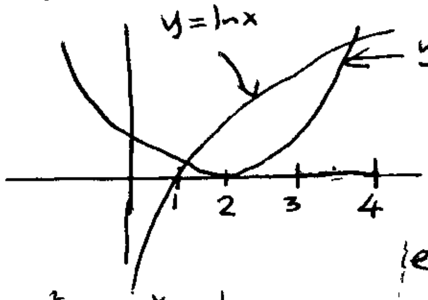
MCS 440 — Prof. Richard B. Goldstein — Burden + Faires 8th Ed — Chapter 1

Sect 1.1

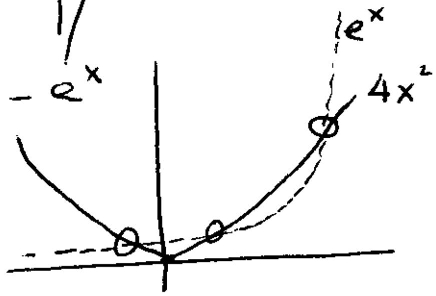
#1b $(x-2)^2 - \ln x = 0$ $[1,2]$ and $[e,4]$



$f(1) = 1 - \ln 1 = +1.00$
$f(2) = 0 - \ln 2 = -0.69$ } sign change

$f(e) = (e-2)^2 - \ln e = -0.48$
$f(4) = 4 - \ln 4 = +2.61$ } sign change

#2b $4x^2 - e^x$



$f(-1) = 4 - e^{-1} = +3.63$
$f(0) = 0 - e^0 = -1.00$ } $\therefore$ $[-1,0]$

$f(0) = -1.0$
$f(1) = 4 - e = +1.28$ } $\therefore$ $[0,1]$
$f(2) = 8.61$
$f(3) = 15.91$
$f(4) = 9.40$
$f(5) = -48.41$ } $\therefore$ $[4,5]$ (this last one is difficult to find)

* #11 $f(x) = (x-1)\ln x$ $x_0 = 1$ $f(1) = 0 \cdot 0 = 0$

(a) $f'(x) = 1 \cdot \ln x + (x-1)\frac{1}{x} = \ln x + \frac{x-1}{x} = \ln x + 1 - \frac{1}{x} \Big|_{x=1} = 0 + 1 - 1 = 0$

$f''(x) = \frac{1}{x} + \frac{1}{x^2} \Big|_{x=1} = 1 + 1 = 2$

$f'''(x) = -\frac{1}{x^2} - \frac{2}{x^3} \Big|_{x=1} = -1 - 2 = -3$

$P_3(x) = 0 + 0x + \frac{2}{2!}(x-1)^2 + \frac{(-3)}{3!}(x-1)^3 = \boxed{(x-1)^2 - \frac{(x-1)^3}{2}}$

$P_3(0.5) = (0.5-1)^2 - \frac{(0.5-1)^3}{2} = 0.25 + 0.0625 = \boxed{0.3125}$

$f(0.5) = (0.5-1)\ln(0.5) = \boxed{0.3466}$ actual error $\boxed{0.0341}$

$f^{iv}(x) = \frac{2}{x^3} + \frac{6}{x^4}$ $f^{iv}(0.5) = 112$, $f^{iv}(1) = 8$ } error max at left end (112)

$R \leq \frac{112}{4!}(0.5-1)^4 = \boxed{0.2916\ldots}$ max est. error

Sect 1.2

#3b $p = 900$ $\frac{|p^* - p|}{|p|} \leq 10^{-3}$ $\frac{|p^* - 900|}{900} \leq 10^{-3} = 0.001$

$|p^* - 900| \leq 0.9$ $\boxed{899.1 \leq p^* \leq 900.9}$

#4c (i) $\left(\frac{1}{3} - \frac{3}{11}\right) + \frac{3}{20} = \frac{139}{660} = \boxed{0.21060\overline{60} = p}$ exact

(ii) 3 digit chop $(0.333 - 0.272) + 0.150 = 0.061 + 0.150 = \boxed{0.211}$ } p^*

(iii) 3 digit round $(0.333 - 0.273) + 0.150 = 0.060 + 0.150 = \boxed{0.210}$

(iv) relative error $\left|\frac{.211 - .210\overline{60}}{.210\overline{60}}\right| = \underline{1.87 \times 10^{-3}}$ $\left|\frac{.210 - .210\overline{60}}{.210\overline{60}}\right| = \underline{2.88 \times 10^{-3}}$

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#5e $\dfrac{\frac{13}{14} - \frac{6}{7}}{2e - 5.4}$ exact value is $1.9535401\ldots$

3-digit rounding $\dfrac{0.929 - 0.857}{5.44 - 5.40} = \dfrac{0.072}{0.04} = 1.80$

absolute error $0.1535\ldots$
relative error $0.0785958\ldots$

#15c use $(-1)^s \, 2^{c-1023}(1+f)$

0 01111111111 0101|0011|0... 0

$+$ 1023 (c) $\frac{5}{16} + \frac{3}{256} + 0 \ldots = 0.32421875$

$1 \cdot 2^0 (1 + 0.32421875) = 1.32421875$

#19a
$$1.130x - 6.990y = 14.20 \qquad ax + by = e$$
$$1.013x - 6.099y = 14.22 \qquad cx + dy = f$$
4-digit rounding

(1) $m = \frac{c}{a} = \frac{1.013}{1.130} = 0.8965$

(2) $d_1 = d - mb = -6.099 - 0.8965(-6.990) = -6.099 + 6.267 = 0.1680$

(3) $f_1 = f - me = 14.22 - 0.8965(14.20) = 14.22 - 12.73 = 1.490$

(4) $y = \frac{f_1}{d_1} = \frac{1.490}{0.1680} = 8.869$

(5) $x = \dfrac{14.20 - (-6.990)(8.869)}{1.130} = \dfrac{14.20 + 61.99}{1.130} = \dfrac{76.19}{1.130} = 67.43$

exact solution is $x = 67.68259\ldots$ $y = 8.9100529\ldots$ (note error in x!)

abs errors 0.25 0.041

Sect 1.3

#6b $\lim_{n \to \infty} \sin\left(\frac{1}{n^2}\right)$

$$\sin\left(\frac{1}{n^2}\right) = \frac{1}{n^2} - \frac{(1/n^2)^3}{3!} + \frac{(1/n^2)^5}{5!} \ldots \qquad O\left(\frac{1}{n^2}\right)$$
$$= \frac{1}{n^2} - \frac{1}{6n^6} + \frac{1}{120n^{10}} - \ldots$$
(error proportional to $1/n^2$)

#7d $\lim_{h \to 0} \frac{1 - e^h}{h} = -1$

$$\frac{1 - \left(1 + h + \frac{h^2}{2} + \frac{h^3}{6} + \ldots\right)}{h} = \frac{-h - \frac{h^2}{2} - \frac{h^3}{6} \ldots}{h} = -1 - \frac{h}{2} - \frac{h^2}{6} \ldots$$

$O(h)$ (error prop. to h)