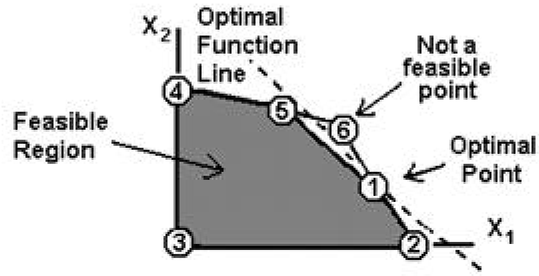
Sensitivity Analysis using a Geometric Perspective - Prof. Richard B. Goldstein

Sensitivity Analysis considers the effect and limitations of changing one parameter in a Linear Programming problem while leaving all of the other parameters unaltered. The standard problem of the form:

Max $c_1x_1 + c_2x_2 + \ldots$
s.t. $a_{11}x_1 + a_{12}x_2 + \ldots <= q_1$
$a_{21}x_1 + a_{22}x_2 + \ldots <= q_2$



considers the perturbation of the objective coefficients (c's) and the right-hand side coefficients (q's).

Objective Function Coefficients

A change in one of these (c_1, c_2, ...) has the effect of rotating the line (or plane in 3-D or higher) while the feasible region remains the same. Any single coefficient may be increased or decreased until the objective line rotates and touches a new corner point.

CALCULATION: Consider what value of the coefficient c_i that would be needed to make the objective function line (or plane) parallel to each edge (or plane) connecting to the optimal point. In the above diagram the dashed line may be rotated until it is parallel to either edge B or C (but not A).
Note: parallel lines or planes have coefficients in a common ratio.

Right-Hand Side Coefficients

A change in one of these (q_1, q_2, ...) has the effect of changing the size of the feasible region by moving one of the edges in a parallel motion either in or out. This movement is done as long as the optimal point remains the intersection of the same set of lines or planes. There are two cases:

(1) Edges through the optimal point: (for example, lines B or C)

CALCULATION: As line B is moved out the optimal point slides up the line connecting points (1) and (6) until it reaches (6). Any further movement of line B outward would keep (6) the optimal point. Moving line B inward would slide the optimal down the line from point (1) to (2). Any further movement of line B inward beyond point (2) would change the nature of the feasible region - the optimal point would no longer be at the intersection of lines B and C. It would start moving from (2) to (3).

(2) Edges not through the optimal point: (for example, line A)

CALCULATION: Line A may be moved out indefinitely (upper limit is infinity) without changing the optimal point. Line A may be moved in until too passes through the optimal point. Then all three lines, A, B, and C go through the optimal point. Any further movement inward changes the optimal from the intersection of lines B and C to lines A and C.

SENSITIVITY ANALYSIS (MIN)

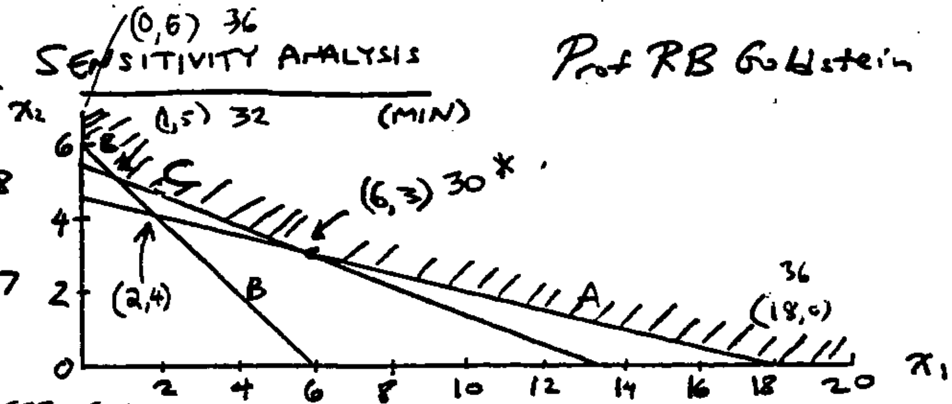
Prof RB Goldstein

Min $2x_1 + 6x_2$

s.t. $x_1 + 4x_2 \geq 18$

$x_1 + x_2 \geq 6$

$2x_1 + 5x_2 \geq 27$



OBJECTIVE FUNCTION COEFFICIENTS

$c_1x_1 + 6x_2$ // A: $x_1 + 4x_2$ $\quad \frac{c_1}{1} = \frac{6}{4} \Rightarrow c_1 = 1.5$

// C: $2x_1 + 5x_2$ $\quad \frac{c_1}{2} = \frac{6}{5} \Rightarrow c_1 = 2.4$

$$\boxed{1.5 \leq \boxed{c_1 = 2} \leq 2.4}$$

$2x_1 + c_2x_2$ // A: $x_1 + 4x_2$ $\quad \frac{2}{1} = \frac{c_2}{4} \Rightarrow c_2 = 8$

// C: $2x_1 + 5x_2$ $\quad \frac{2}{2} = \frac{c_2}{5} \Rightarrow c_2 = 5$

$$\boxed{5 \leq \boxed{c_2 = 6} \leq 8}$$

RIGHT HAND SIDE COEFFICIENTS

(A) $x_1 + 4x_2 \geq b_1$ IN TO (13.5,0) $13.5 + 4(0) = 13.5$

OUT TO (1,5) $1 + 4(5) = 21$

$$\boxed{13.5 \leq \boxed{b_1 = 18} \leq 21}$$

(B) $x_1 + x_2 \geq b_2$ IN no limit

OUT TO (6,3) $6 + 3 = 9$

$$\boxed{-\infty < \boxed{b_2 = 6} \leq 9}$$

(C) $2x_1 + 5x_2 \geq b_3$ IN TO (2,4) $2(2) + 5(4) = 24$

OUT TO (18,0) $2(18) + 6(0) = 36$

$$\boxed{24 \leq \boxed{b_3 = 27} \leq 36}$$

SHADOW PRICES

$x_1 + 4x_2 = 18 \Rightarrow x_1 + 4x_2 = 17 \Rightarrow$ new solution $\left(\frac{23}{3}, \frac{7}{3}\right)$ MIN $= \frac{88}{3} = 29.33$

shadow price $= 30 - 29.33 = \underline{0.67}$

$2x_1 + 5x_2 = 27 \Rightarrow 2x_1 + 5x_2 = 26 \Rightarrow$ new solution $\left(\frac{14}{3}, \frac{10}{3}\right)$ MIN $= \frac{88}{3} = 29.33$

shadow price $= 30 - 29.33 = \underline{0.67}$

COMPUTER OUTPUT: MANAGEMENT SCIENTIST 5.0

LINEAR PROGRAMMING PROBLEM

MIN 2X1+6X2

S.T.

1) 1X1+4X2>18
2) 1X1+1X2>6
3) 2X1+5X2>27

OPTIMAL SOLUTION

Objective Function Value = 30.000

Variable	Value	Reduced Costs
X1	6.000	0.000
X2	3.000	0.000

Constraint	Slack/Surplus	Dual Prices
1	0.000	-0.667
2	3.000	0.000
3	0.000	-0.667

OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
X1	1.500	2.000	2.400
X2	5.000	6.000	8.000

RIGHT HAND SIDE RANGES

Constraint	Lower Limit	Current Value	Upper Limit
1	13.500	18.000	21.000
2	No Lower Limit	6.000	9.000
3	24.000	27.000	36.000