MBA 515 – Prof. Richard B. Goldstein – HW Answers – Anderson – 12th Ed

Chapter 2

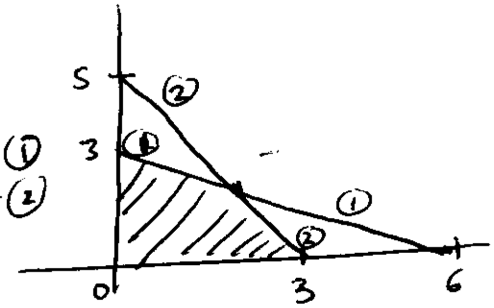
#10 Max $2A+3B$

s.t. $1A+2B \le 6$ (1)

$5A+3B \le 15$ (2)

$A,B \ge 0$

$5A+10B=30$
$5A+3B=15$
$7B=15 \Rightarrow B=\frac{15}{7}$ & $A=6-2(\frac{15}{7})=\frac{12}{7}$



A	B	Obj
0	0	0
3	0	6
12/7	15/7	69/7 = 9.857
0	3	9

max at $(\frac{12}{7}, \frac{15}{7})$ of $\frac{69}{7}$

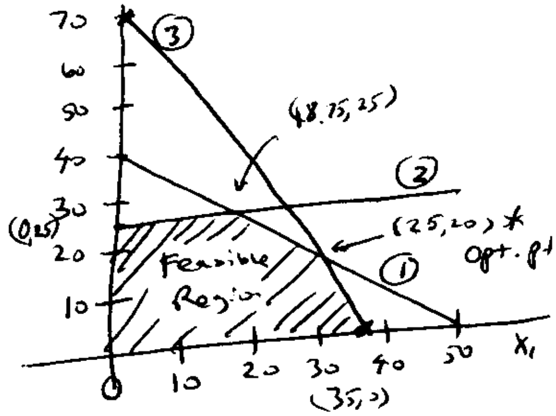
#14 (a) let x_1 = # tons of fuel additive
x_2 = # tons of solvent base

Max $40x_1+30x_2$

s.t. $0.4x_1+0.5x_2 \le 20$ (material #1)

$0.2x_2 \le 5$ #2

$0.6x_1+0.3x_2 \le 21$ #3



(b)

x_1	x_2	$40x_1+30x_2$
35	0	1400
25	20	1600 *
18.75	25	1500
0	25	750

(c) material #2
4 tons are used
and 1 ton is unused

(d) there are no redundant constraints (if any constraint is removed the feasible region is changed)

#35 Min $6A+4B$

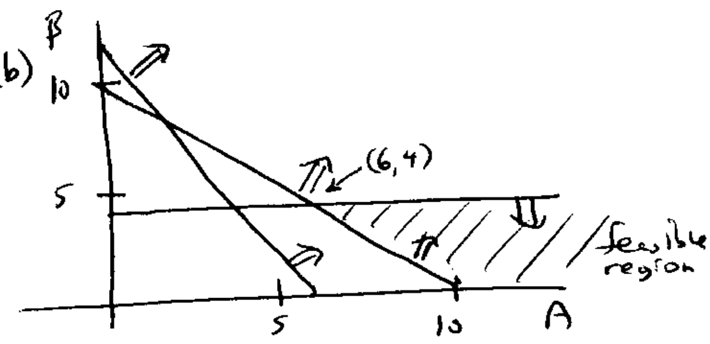
s.t. $2A+1B \ge 12$
$1A+1B \ge 10$
$1B \le 4$

$A,B \ge 0$

$\Rightarrow$ (a) Min $6A+4B$

s.t. $2A+1B-S_1=12$
$1A+1B-S_2=10$
$1B+S_3=4$

(b)



A	B	6A+4B
10	0	60
6	4	52 *

(c) $2(6)+1(4)-S_1=12$ surplus $S_1=4$
$(6)+(4)-S_2=10$ surplus $S_2=0$
$(4)+S_3=4$ slack $S_3=0$

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#37 Regular Blend : 80% mild cheddar 20% extra sharp
Zesty : 60% " " 40% " "

Available – 8,100 lbs. mild cheddar @ $1.20 / lb
– 3,000 lbs. extra sharp @ 1.40 / lb.

Cost – blend & package @ $0.20 / container ≡ 12 oz ← $\frac{3}{4}$ lb

Revenue – Regular $1.95/container, Zesty $2.20 / container

<u>Cost</u> : 1 container of Regular Blend $\frac{3}{4}[0.8(1.20) + 0.2(1.40)] + 0.2 = \$1.13$

" of Zesty $\frac{3}{4}[0.6(1.20) + 0.4(1.40)] + 0.2 = 1.16$

<u>Profit</u> Regular $1.95 - 1.13 = \underline{0.82}$ Zesty $2.20 - 1.16 = \underline{1.04}$

$$\text{Max } 0.82R + 1.04Z$$

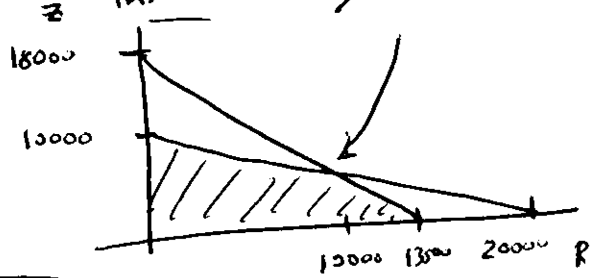
s.t. $0.75[0.8R + 0.6Z] \le 8100 \Rightarrow 0.60R + 0.45Z \le 8100$

$0.75[0.2R + 0.4Z] \le 3000 \Rightarrow 0.15R + 0.30Z \le 3000$

$R, Z \ge 0$

<u>intersect at</u> (9,600, 5200)

R	Z	Profit = 0.82R + 1.04Z
0	0	0
13,500	0	$11,070
9600	5,200	13,280 ⟸ opt.
0	10,000	10,400



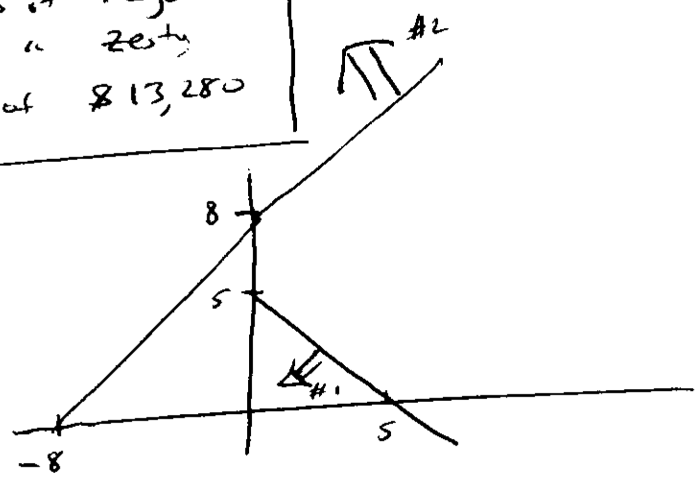
9,600 containers of regular
& 5,200 " " zesty
for a profit of $13,280

#42 Max $4A + 8B$

s.t. $2A + 2B \le 10$

$-1A + 1B \ge 8$

$A, B \ge 0$



this is an <u>infeasible</u> problem

Chapter 3

#12 Problem:

$$\begin{aligned} \text{Max } & 63E + 95S + 135D \\ \text{s.t. } & 1E + 1S + 1D \le 200 \quad \text{Fan Motors} \\ & 1E + 2S + 4D \le 320 \quad \text{Cooling Coils} \\ & 8E + 12S + 14D \le 2400 \quad \text{Manufacturing time} \\ & E, S, D \ge 0 \end{aligned}$$

(a) from output $E = 80$, $S = 120$, $D = 0$ & Profit = \$16,440

(b) there is no slack/surplus for constraints #1 (Fan Motors)

nor #2 (cooling fans) ∴ those are binding

(c) Constraint #3 (manufacturing time) has

$2400 - 8(80) - 12(120) - 14(0) = 2400 - 640 - 1440 = 320$ hours

(d) Coeff of D: $135 \rightarrow 150$ Limits of D: $\{D = 135\} \le 159$

This implies that the solution is the same for any value ≤ 159

#13 (a)

E: $47.5 \le \{C_E = 63\} \le 75$

S: $87.0 \le \{C_S = 95\} \le 126$

D: $\{C_D = 135\} \le 159$

(b)

E: $63 \rightarrow 69$ $\frac{6}{12} = 50\%$

S: $95 \rightarrow 93$ $\frac{2}{8} = 25\%$

D: $135 \rightarrow 139$ $\frac{4}{24} = 16.7\%$

adds to $91.7\% < 100\%$ ∴ OK same answer

(c) RHS

$160 \le \{b_1 = 200\} \le 280$

$200 \le \{b_2 = 320\} \le 400$

$2{,}080 \le \{b_3 = 2400\}$

(d) RHS b_1: $200 \rightarrow 300$ exceeds upper bound of 280

∴ dual price changes

#29 Let x_1 = # gallons of white wine
x_2 = # gallons of rose wine
x_3 = # gallons of fruit juice

(a) the cost of wine and fruit juice are relevant costs because these items can be purchased as needed

(b) Max $(2.50-1.00)x_1 + (2.50-1.50)x_2 + (2.50-0.5)x_3 = 1.5x_1 + 1x_2 + 2x_3$

s.t. $\frac{x_1}{x_1+x_2+x_3} \geq 0.5 \Rightarrow 0.5x_1 - 0.5x_2 - 0.5x_3 \geq 0$ % white

$0.2 \leq \frac{x_2}{x_1+x_2+x_3} \leq 0.3 \Rightarrow \left.\begin{array}{l} -0.2x_1 + 0.8x_2 - 0.2x_3 \geq 0 \\ -0.3x_1 + 0.7x_2 - 0.3x_3 \leq 0 \end{array}\right\}$ % rose

$\frac{x_3}{x_1+x_2+x_3} = 0.2 \Rightarrow -0.2x_1 - 0.2x_2 + 0.8x_3 = 0$ % fruit juice

$x_1 \leq 10{,}000$

$x_2 \leq 8{,}000$

[based upon results on next page]

Optimal solution $\left.\begin{array}{l} x_1 = 10{,}000 \\ x_2 = 6{,}000 \\ x_3 = 4{,}000 \end{array}\right\}$ + profit = \$29,000

(c) Yes – normal cost is \$1/gal. – Marginal profit is \$2.90/gal therefore they would be willing to pay up to 1.00 + 2.90 = \$3.90/gal for addition gallons

(d) no – they don't even use currently all available rose gallons

(e)+(f) more complicated – because % changes problem left hand side in constraints

LINEAR PROGRAMMING PROBLEM - Chapter 3 #29

MAX 1.5X1+1X2+2X3

S.T.

1) 0.5X1-0.5X2-0.5X3>0
2) -0.2X1+0.8X2-0.2X3>0
3) -0.3X1+0.7X2-0.3X3<0
4) -0.2X1-0.2X2+0.8X3=0
5) 1X1<10000
6) 1X2<8000

OPTIMAL SOLUTION

Objective Function Value = 29000.000

Variable	Value	Reduced Costs
X1	10000.000	0.000
X2	6000.000	0.000
X3	4000.000	0.000

Constraint	Slack/Surplus	Dual Prices
1	0.000	0.000
2	2000.000	0.000
3	0.000	2.400
4	0.000	3.400
5	0.000	2.900
6	2000.000	0.000

OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
X1	-1.400	1.500	No Upper Limit
X2	-0.500	1.000	No Upper Limit
X3	-4.000	2.000	No Upper Limit

RIGHT HAND SIDE RANGES

Constraint	Lower Limit	Current Value	Upper Limit
1	No Lower Limit	0.000	0.000
2	No Lower Limit	0.000	2000.000
3	-1666.667	0.000	0.000
4	-2857.143	0.000	0.000
5	0.000	10000.000	13333.333
6	6000.000	8000.000	No Upper Limit

Chapter 4

#4 Quality Assurance

(a) let x_1 = # lbs. of bean 1
x_2 = # lbs of bean 2
x_3 = # lbs of bean 3

Min $0.5x_1 + 0.7x_2 + 0.45x_3$

s.t. $\frac{75x_1 + 85x_2 + 60x_3}{x_1 + x_2 + x_3} \geq 75 \Rightarrow 10x_1 \quad - 15x_3 \geq 0$ Aroma

wt. avg.

$\frac{86x_1 + 88x_2 + 75x_3}{x_1 + x_2 + x_3} \geq 80 \Rightarrow 6x_1 + 8x_2 - 5x_3 \geq 0$ Taste

$x_1 \leq 500$
$x_2 \leq 600$
$x_3 \leq 400$
$x_1 + x_2 + x_3 = 1000$

Optimal Solution $x_1 = 500 \quad x_2 = 300 \quad x_3 = 200$ Cost = \$550

(b) Cost per pound 550/1000 = \$0.55

(c) Aroma $\frac{75(500) + 85(300) + 60(200)}{500 + 300 + 200} = \frac{75000}{1000} = 75$ meets minimum exactly

Taste $\frac{86(500) + 88(300) + 75(200)}{500 + 300 + 200} = \frac{84400}{1000} = 84.4 > 80$ exceeds taste min.

(d) dual price of last eq. $(x_1 + x_2 + x_3 = 1000)$ gives 60¢/lb.

#15 Let x_{1R} = gallons of crude 1 used to produce regular
x_{1H} = " " " " " " " high octane
x_{2R} = " " " 2 " " " regular
x_{2H} = " " " " " " " high octane

Min $0.10x_{1R} + 0.10x_{1H} + 0.15x_{2R} + 0.15x_{2H}$

s.t. amt of A in regular $\geq$ amt. of A required for regular

$0.2x_{1R} + 0.5x_{2R} \geq 0.4x_{1R} + 0.4x_{2R} \Rightarrow -0.2x_{1R} + 0.1x_{2R} \geq 0$

amt of B in high octane $\leq$ amt. of B required for high octane

$0.6x_{1H} + 0.3x_{2H} \leq 0.5x_{1H} + 0.5x_{2H} \Rightarrow 0.1x_{1H} - 0.2x_{2H} \leq 0$

$x_{1R} + x_{2R} \geq 800{,}000$ available
$x_{1H} + x_{2H} \geq 500{,}000$

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LINEAR PROGRAMMING PROBLEM - Chapter 4 #4

MIN 0.5X1+0.7X2+0.45X3

S.T.

1) 10X2-15X3>0
2) 6X1+8X2-5X3>0
3) 1X1<500
4) 1X2<600
5) 1X3<400
6) 1X1+1X2+1X3=1000

OPTIMAL SOLUTION

Objective Function Value = 550.000 ← Cost

Variable	Value		Reduced Costs
X1	500.000	← bean #1	0.000
X2	300.000	#2	0.000
X3	200.000	#3	0.000

Constraint	Slack/Surplus	Dual Prices
1	0.000	-0.010
2	4400.000	0.000
3	0.000	0.100
4	300.000	0.000
5	200.000	0.000
6	0.000	-0.600

OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
X1	No Lower Limit	0.500	0.600
X2	0.533	0.700	No Upper Limit
X3	0.200	0.450	0.700

RIGHT HAND SIDE RANGES

Constraint	Lower Limit	Current Value	Upper Limit
1	-5000.000	0.000	5000.000
2	No Lower Limit	0.000	4400.000
3	0.000	500.000	1000.000
4	300.000	600.000	No Upper Limit
5	200.000	400.000	No Upper Limit
6	500.000	1000.000	1500.000

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LINEAR PROGRAMMING PROBLEM - Chapter 4 #15

MIN 0.1X1R+0.1X1H+0.15X2R+0.15X2H

S.T.

1) -0.2X1R+0.1X2R>0
2) 0.1X1H-0.2X2H<0
3) 1X1R+1X2R>800000
4) 1X1H+1X2H>500000

OPTIMAL SOLUTION

Objective Function Value = 165000.000 ⇐ minimum cost

Variable	Value		Reduced Costs
X1R	266666.667 ⇐	crude1 regular	0.000
X1H	333333.333	crude 1 high	0.000
X2R	533333.333	crude2 regular	0.000
X2H	166666.667	crude 2 high	0.000

Constraint	Slack/Surplus	Dual Prices
1	0.000	-0.167
2	0.000	0.167
3	0.000	-0.133
4	0.000	-0.117

OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
X1R	-0.300	0.100	0.150
X1H	-0.075	0.100	0.150
X2R	0.100	0.150	No Upper Limit
X2H	0.100	0.150	No Upper Limit

RIGHT HAND SIDE RANGES

Constraint	Lower Limit	Current Value	Upper Limit
1	-160000.000	0.000	80000.000
2	-100000.000	0.000	50000.000
3	0.000	800000.000	No Upper Limit
4	0.000	500000.000	No Upper Limit

Chapter 13

#2 (a)

decision	row max	row min
d_1	14*	5
d_2	11	7
d_3	11	9*
d_4	13	8

optimistic: d_1 (14) - maximax
pessimistic: d_3 (9) - maximin

Regret/Opportunity Loss Table

	S_1	S_2	S_3	S_4	row max
d_1	0	1	1	8	8
d_2	3	0	3	6	6
d_3	5	0	1	2	5* minimax
d_4	6	0	0	0	6

(b) Choice varies with decision maker - the most appropriate approach should be selected before analyzing the problem

* (c) based upon cost

decision	min cost	max cost
d_1	5 *	14
d_2	7	11 *
d_3	9	11 *
d_4	8	13

optimist d_1 (5)   pessimist d_2, d_3 tie (11)

Reg/opt Loss

6	0	2	0	6
3	1	0	2	3 * d_2 selected
1	1	2	6	
0	1	3	8	

* note: cost is not on exams (just profit or revenue decisions)

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#9 (a) decision : choose type of service
Chance event : level of demand
Consequence : amount of qtrly. profit
alt. decisions : full or discount price
outcomes : strong or weak demand

(b)

Type of Service	max. profit	min. profit
full price	\$ 960	− \$ 490
discount	\$ 670	\$ 320

Opt. : full price service
Pess. : discount service

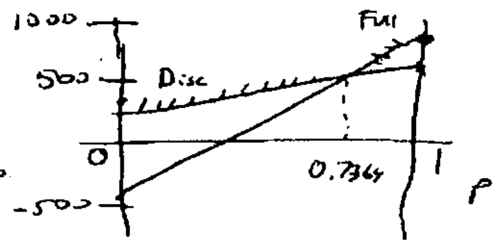
Opp. Loss / Regret Table

	high	low	max regret
Full	0	810	810
Discount	290	0	290 ⟸ choose this

(c) $EV(full) = 0.7(960) + 0.3(-490) = 525$
$EV(discount) = 0.7(670) + 0.3(320) = 565$ ⟸ discount

(d) $EV(full) = 0.8(960) + 0.2(-490) = 670$ ⟸ full price
$EV(discount) = 0.8(670) + 0.2(320) = 600$

(e) let p = prob.{strong demand}
$EV(full) = p(960) + (1-p)(-490) = 1450p - 490$
$EV(discount) = p(670) + (1-p)(320) = 350p + 320$
$1450p - 490 = 350p + 320$
$1100p = 810$
$p = 810/1100 = \underline{\underline{0.7364}}$



for $p < 0.7364 \Rightarrow$ discount
$p > 0.7364 \Rightarrow$ full

#14 (a) if s_1 then d_1
if s_2 then d_1 or d_2
if s_3 then d_2

	s_1	s_2	s_3
d_1	250	100	25
d_2	100	100	75

(b) $EVwPI = 0.65(250) + 0.15(100) + 0.20(75) = 192.5$

(c) from the solution to problem #5 we know that
$EV(d_1) = 182.5$ and $EV(d_2) = 95$
$\therefore$ the recommended decision is d_1 $EVwoPI = 182.5$

(d) $EVPI = EVwPI - EVwoPI = 192.5 - 182.5 = 10$

(11)

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Chapter 14 – Problem #25 – *Prof. Richard B. Goldstein*

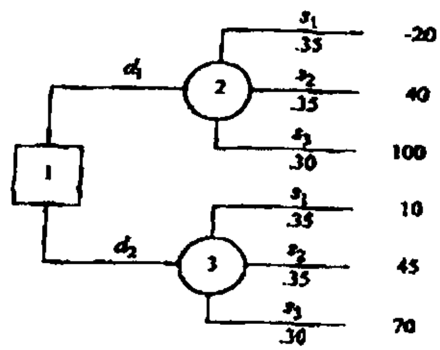
	State of Nature		
Decision Alternative	Low Demand s_1	Medium Demand s_2	High Demand s_3
Manufacture, d_1	-20	40	100
Purchase, d_2	10	45	70

a.

EV(node 2) = 0.35(-20) + 0.35(40) + 0.30(100) = 37

EV(node 3) = 0.35(10) + 0.35(45) + 0.30(70) = 40.25

Recommended decision: d_2 (purchase component)



b. Optimal decision strategy with perfect information:

If s_1, then d_2 If s_2, then d_2 If s_3, then d_1

EVwPI = 0.35(10) + 0.35(45) + 0.30(100) = 49.25
EVoPI = 49.25 – 40.25 = 9 (which is $9,000)

c. If F (Favorable)

State of Nature	$P(s_j)$	$P(F \mid s_j)$	$P(F \cap s_j)$	$P(s_j \mid F)$
s_1	0.35	0.10	0.035	0.0986
s_2	0.35	0.40	0.140	0.3944
s_3	0.30	0.60	0.180	0.5070
		P(F) =	0.355	

If U (Unfavorable)

State of Nature	$P(s_j)$	$P(U \mid s_j)$	$P(U \cap s_j)$	$P(s_j \mid U)$
s_1	0.35	0.90	0.315	0.4884
s_2	0.35	0.60	0.210	0.3256
s_3	0.30	0.40	0.120	0.1860
		P(U) =	0.645	

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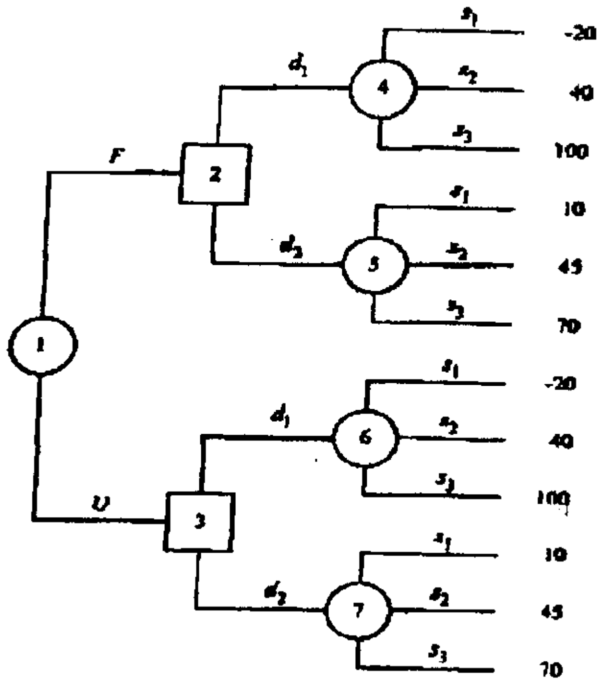
d. Expected value of nodes:

4	64.51
5	54.23
6	21.86
7	32.56



Decision strategy

If F then d_1 since EV(4) > EV(5)

If U then d_2 since EV7) > EV(6)

EV(node 1) = 0.355(64.51) + 0.645(32.56)
= 43.90

e. EVwPI = 49.25
EVwSI = 43.90

EV(d_1) = 37
EV(d_2) = 40.25

+ f,

Expected Value with no information = 40.25

EVoSI = 43.90 – 40.25 = 3.65

Efficiency = 3.65/9.00 = 0.406 or 40.6%

EVwoPI EVwSI EVwPI
43.90
40.25 49.25
3.65
9.00

$\frac{3.65}{9.00} = 40.6\%$

Utility Theory (from 11th Ed)

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A firm has three investment alternatives. The payoff table (in thousands of dollars) and associated probabilities are as follows:

	Economic Condition		
Investment	Up	Stable	Down
d_1	100	25	0
d_2	75	50	25
d_3	50	50	50
Probabilities	0.40	0.30	0.30

a. Using the expected value approach, which decision is preferred?

b. For the lottery payoff of \$100,000 with probability p and \$0 with probability (1 – p), two decision makers expressed the following indifference probabilities:

	Indifference Probability (p)	
Profit	Decision Maker A	Decision Maker B
\$75,000	0.80	0.60
50,000	0.60	0.30
25,000	0.30	0.15

Find the preferred decision for each decision maker using the expected utility approach.

(a) $E(d_1) = 0.40(100) + 0.30(25) + 0.30(0) = 47.5$

$E(d_2) = 0.40(75) + 0.30(50) + 0.30(25) = 52.5$ * select d_2

$E(d_3) = 0.40(50) + 0.30(50) + 0.30(50) = 50.0$

(b) Using utility values (replace 0 to 100 by 0–10 as given)

Decision Maker A:

$EU(d_1) = 0.40(10.0) + 0.30(3.0) + 0.30(0) = 4.9$

$EU(d_2) = 0.40(8.0) + 0.30(6.0) + 0.30(3.0) = 5.9$

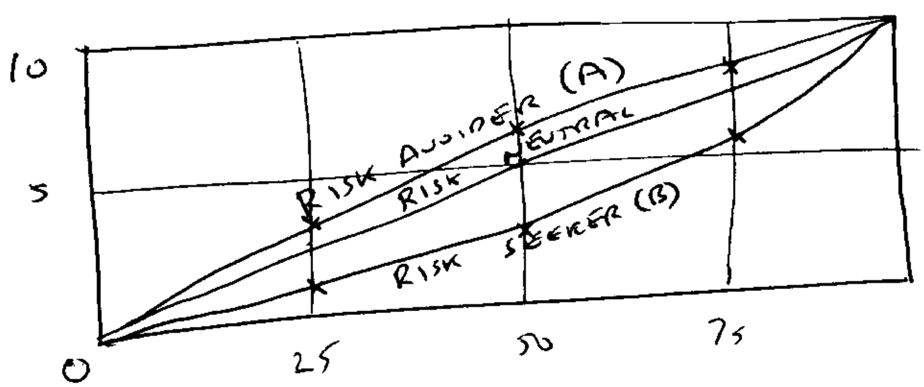
$EU(d_3) = 0.40(6.0) + 0.30(6.0) + 0.30(6.0) = 6.0 \Leftarrow$ select d_3

Decision Maker B:

$EU(d_1) = 0.4(10.0) + 0.3(1.5) + 0.3(0) = 4.45 \Leftarrow$ select d_1

$EU(d_2) = 0.4(6.0) + 0.3(3.0) + 0.3(1.5) = 3.75$

$EU(d_3) = 0.4(3.0) + 0.3(3.0) + 0.3(3.0) = 3.0$



GAME THEORY - Chap 5

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#14

		D: b_1 Indianapolis	b_2 Evansville	b_3 Ft. Wayne	b_4 South Bend	row min
R	a_1	0	−15	−8	20	−15
	a_2	30	−5	5	−10	−10
	a_3	10	−25	0	20	−25
	⇒ a_4	20	20	10	15	10
	col max	30	20	10	20	

pure strategies a_4 & b_3 ⇒ value = 10

#15 Mixed Strategy

Player A (rows), player B (columns)

	R	W	B	
R	0	−1	2	−1
W	5	4	−3	−3
B	2	3	−4	−4
	5	4	2	

W > B for player A (elim B)

	R	W	B
R	0	−1	2
W	5	4	−3

R > W (player B eliminates R)

		W	B
p	R	−1	2
$1-p$	W	4	−3
		q	$1-q$

for player B

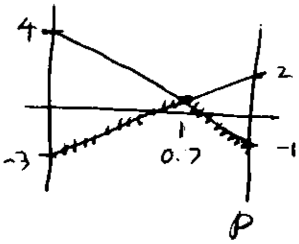
$E_W = (-1)p + 4(1-p) = 4-5p$

$E_B = 2p + (-3)(1-p) = -3+5p$

$4-5p = -3+5p$

$7 = 10p \quad p = 0.7$

$V = 4-5(0.7) = \underline{\underline{0.5}} = -3+5(0.7)$



for player A

$E_R = (-1)q + 2(1-q) = 2-3q$

$E_W = 4q + (-3)(1-q) = -3+7q$

$2-3q = -3+7q$

$5 = 10q$

$q = 0.5$

$V = 2-3(0.5) = \underline{\underline{0.5}} = -3+7(0.5)$