LINES – Graphing, Intersections & Inequalities – **Prof. Richard B. Goldstein**

Lines

Form: y = mx + b

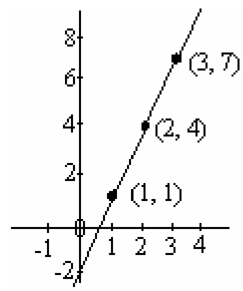
To graph either (1) make a table of x & y, plot three points, & connect

or (2) show the y-intercept and slope

Example:

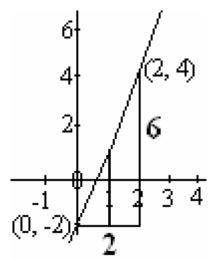
y = 3x - 2 table:

x	y
1	1
2	4
3	7



y-intercept (0, -2)

slope (rise) 3 = 6/2



Form: Ax + By = C

To graph find interceptions (alternately set x = 0 and y = 0) and connect: $\left(0, \frac{C}{B}\right)$ and $\left(\frac{C}{A}, 0\right)$

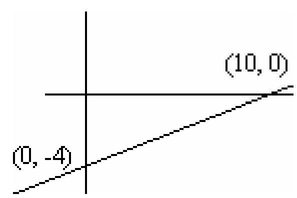
Note: If C = 0, one point is the origin (0, 0) and another can be found be setting x = 1.

Example:

2x – 5y = 20

set x = 0 -5y = 20 yields (0, -4)

set y = 0 2x = 20 yields (10, 0)



Intersections

Two lines may (1) intersect once **example:** 2x + 3y = 8 intersect at (1, 2)

3x – 4y = -5

(2) coincide **example:** 2x + 3y = 8

6x + 9y = 24

(3) be parallel **example:** 2x + 3y = 8

4x + 6y = 12

Lines, and also planes (3D) and hyper-planes (higher dimension) *coincide* if all of the coefficients and the right hand side of the equation have a constant ratio: $\frac{2}{6}=\frac{3}{9}=\frac{8}{24}$ as in the above example

Lines, etc. are *parallel* if all of the coefficients have a constant ratio but the right hand side ratio is different: $\frac{2}{4}=\frac{3}{6}\neq\frac{8}{12}$

Otherwise, common methods for solving for the *intersection*:

(1) eliminate one of the variables

(2) substitution

(3) Gauss' method (or Gauss-Jordan, etc.)

Example:

(1) $2x + 3y = 8$ (multiply by 4) $8x + 12y = 32$

$3x - 4y = -5$ (multiply by 3) $9x - 12y = -15$

adding and eliminating y $17x = 17$ yields $x = 1$

substituting $2(1) + 3y = 8$ yields $y = 2$

(2) $2x + 3y = 8$ becomes $3y = 8 - 2x$ or $y = \frac{8}{3}-\frac{2}{3}x$

Substituting in the second equation: $3x - 4\left(\frac{8}{3}-\frac{2}{3}x\right) = -5$ simplifies to

$\frac{17}{3}x-\frac{32}{3}=-5$ or $\frac{17}{3}x=\frac{17}{3}$ which yields $x = 1$ and

$y = \frac{8}{3}-\frac{2}{3}(1) = 2$

(3) $\left[\begin{array}{cc|c} 2 & 3 & 8 \\ 3 & -4 & -5 \end{array}\right]$ multiply row #1 by -1.5 and add to row #2: $\left[\begin{array}{cc|c} 2 & 3 & 8 \\ 0 & -8.5 & -17 \end{array}\right]$

Now row #2 can be read as $-8.5y = -17$ which yields $y = 2$

Substitute y in the first equation and solve for x. Note that this method or similar methods are needed for larger systems (several linear equations with several unknowns).

<u>Inequalities</u>

Ax + By ≤ C or Ax + By ≥ C

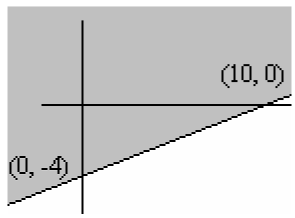
(note that < or > without the = are rare in Operations Research)

Start by graphing the line Ax + By = C as given above. Use the origin, (0, 0) as the test point.

Example:

For 2x – 5y ≤ 20 the test point, the origin, satisfies the equation: 2(0) – 5(0) = 0 < 20

Therefore, the origin and all points on that side of the line satisfy the inequality:

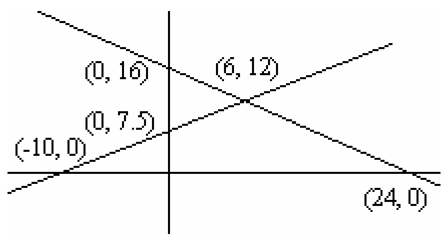


Example:

For two or more equations start by plotting the lines and finding the intersection(s)

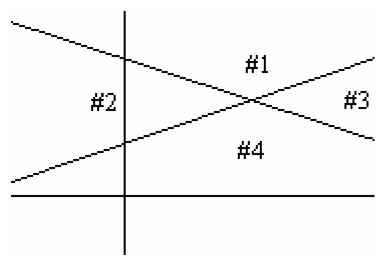
2x + 3y ≤ 48

3x – 4y ≥ -30

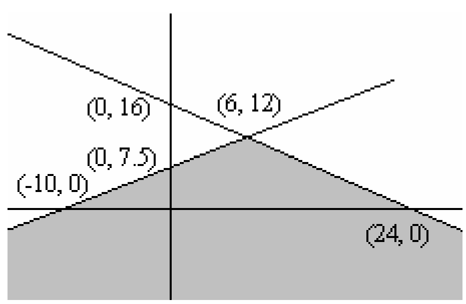


Next, by testing the origin, both equations are satisfied.

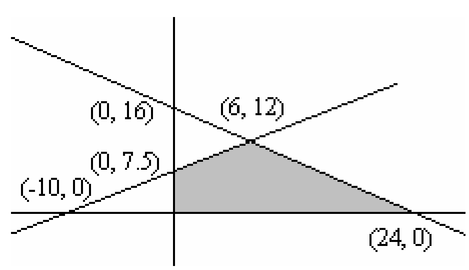
Therefore of the four possible regions only #4 satisfies both:



We now have:

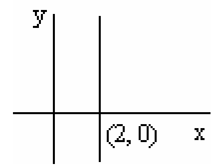


If the inequalities (as in Operations Research) include x ≥ 0 and y ≥ 0 we stay only in the first quadrant:

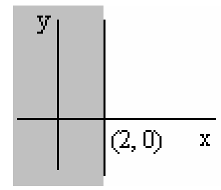


Further considerations

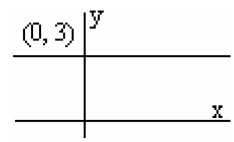
x = k is a vertical line through (k, 0) **example**: x = 2



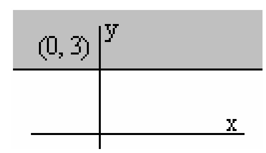
$x \leq k$ is shown shaded to the left **example**: $x \leq 2$



y = k is a horizontal line through (0, k) **example**: y = 3

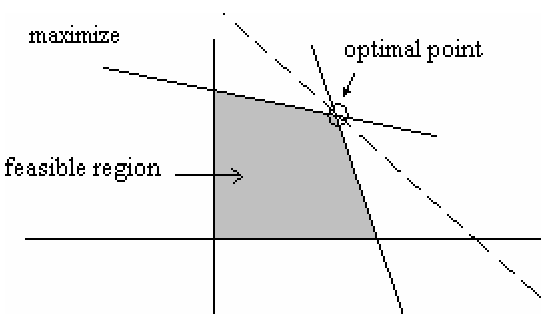


$y \geq 3$ is shown shaded above **example**: $y \geq 3$

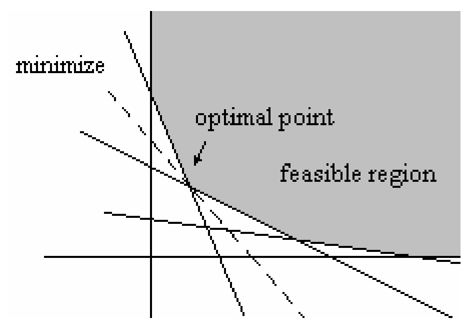


Operations Research equations usually use subscripted variables: x_1, x_2, x_3, etc.

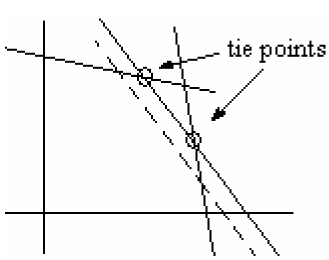
Typical two – variable problems:

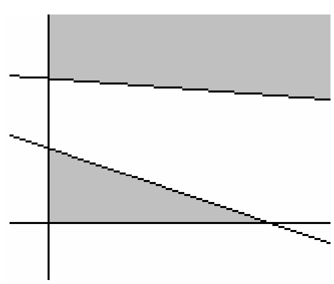


the above region is bounded

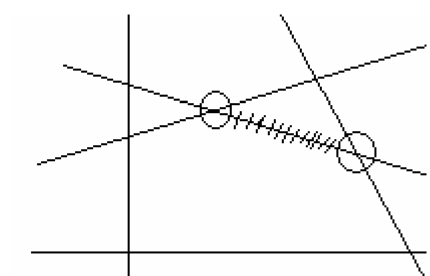


the above region is unbounded





infeasible region (no solution)



equality with the solution on the endpoints of a line segment