USING LINEAR PROGRAMMING IN GAME THEORY

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Player #1	Player #2 (A)	(B)	(C)
(1)	0	1	4
(2)	2	4	-5
(3)	3	5	-3
(4)	-2	0	-4

(1) Player #1 strategy: Choose strategy (1), (2), (3), or (4) w.p. p_1, p_2, p_3, and p_4

Solve the following linear programming problem:

$$\begin{aligned} \text{MAX} \quad & 0p_1 + 0p_2 + 0p_3 + 0p_4 + V \\ \text{s.t.} \quad & 2p_2 + 3p_3 - 2p_4 - V \geq 0 \\ & p_1 + 4p_2 + 5p_3 - V \geq 0 \\ & 4p_1 - 5p_2 - 3p_3 - 4p_4 - V \geq 0 \\ & p_1 + p_2 + p_3 + p_4 = 1 \quad * \end{aligned}$$

The solution is $p_1 = 0.6$, $p_2 = 0$, $p_3 = 0.4$, $p_4 = 0$, and $V = 1.2$.

(2) Player #2 strategy: Choose (A), (B), or (C) w.p. p_A, p_B, and p_C.

Solve the following linear programming problem:

$$\begin{aligned} \text{MIN} \quad & 0p_A + 0p_B + 0p_C + V \\ \text{s.t.} \quad & p_B + 4p_C - V \leq 0 \\ & 2p_A + 4p_B - 5p_C - V \leq 0 \\ & 3p_A + 5p_B - 3p_C - V \leq 0 \\ & -2p_A - 4p_C - V \leq 0 \\ & p_A + p_B + p_C = 1 \quad * \end{aligned}$$

The solution is $p_A = 0.7$, $p_B = 0$, $p_C = 0.3$, and $V = 1.2$.

(C) In the equations starred (*) above, if 1 was changed to 1000, then the values of p and V would be 1000 times larger. This could be used to get more accuracy.

(D) It is important that V, the value of the game, be positive. This could be accomplished by adding a fixed amount to every entry in the table. Then, the probabilities would be the same, but the value of the game would be increased by that fixed amount. For example, if every entry in the game had 6 added on, then all of the coefficients would be positive and V would be 7.2 (which is 1.2 + 6).

(E) It is safer to eliminate dominated rows and columns. Eliminate rows (2) and (4) and column (B) and work with the reduced 2 X 2 matrix of rows (1) & (3) and columns (A) & (C).

Game Theory Simulation

Here is a little on-line Javascript utility for game theory (up to five strategies for the row and column player). It is also designed to play against you (using the optimal mixed strategy most of the time...)

Notes:

- This will only work on Netscape or Internet Explorer, version 3 or later.
- You need only enter the non-zero payoffs. The software will set the others to zero.
- To play against the computer, enter the payoffs, press "Play" and click on row strategies. (The computer does not know your move...)

Payoff Matrix

		COMPUTER				
		1	✗	3	✗	✗
YOU	1	0	1	4		
	✗	2	4	-5		
	3	3	5	-3		
	✗	-2	0	-4		
	✗					

SETTINGS: Show Row Strategy ☑ Show Column Strategy ☑

ACTION: Reduce by Dominance | Check for Saddle Points | Solve Game | Play Game | Stop Play | Erase Everything

STATUS:

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Dominated (or empty) rows and columns have been marked
with an 'x'.
The optimal column strategy is:
[0.7 0 0.3 0 0 ]
The optimal row strategy is:
[0.6 0 0.4 0 0 ]
The value of this game is 1.2.
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