Math 417 - Prof. Richard B. Goldstein

$f(x) = k(\pi^2 x - x^3)$ on $[0, \pi]$ expanded as a half-interval sine series

$$B_n = \frac{2k}{\pi}\int_0^{\pi}(\pi^2 x - x^3)\sin nx\,dx = \frac{2k}{\pi}\left[-\frac{(\pi^2 x - x^3)}{n}\cos nx + \frac{(\pi^2 - 3x^2)}{n^2}\sin nx + \frac{(-6x)}{n^3}\cos nx - \frac{(-6)}{n^4}\sin nx\right]_0^{\pi}$$

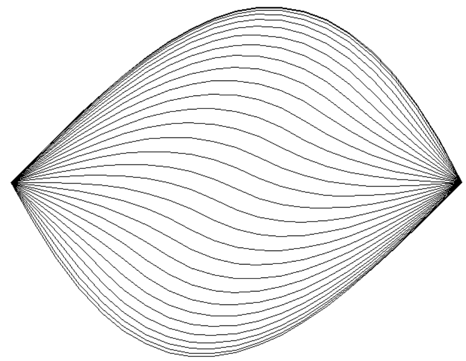
$$B_n = \frac{2k}{\pi}\left[(0 + 0 + \frac{(-6\pi)(-1)^n}{n^3} + 0) - (0 + 0 + 0 + 0)\right] = -\frac{12(-1)^n}{n^3}$$

$$f(x) \sim 12k\left(\sin x - \frac{\sin 2x}{2^3} + \frac{\sin 3x}{3^3} - \frac{\sin 4x}{4^3} + \ldots\right)$$

VIBRATING STRING

C = 0.1, L = π, f(x) = k(π²x -x³), g(x) =0

$$\lambda_n = \frac{nc\pi}{L} = \frac{n(0.1)\pi}{\pi} = \frac{n}{10}$$

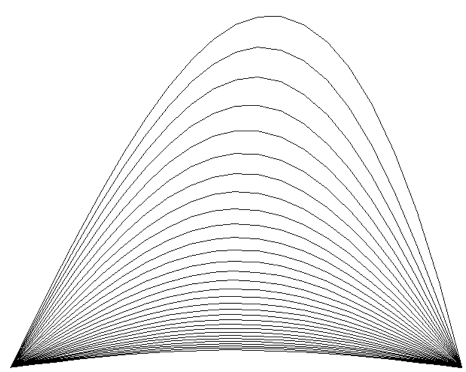


$$\sum_{n=1}^{\infty} B_n \cos\left(\frac{nt}{10}\right)\sin(nx) = 12k\left(\cos\left(\frac{t}{10}\right)\sin x - \frac{\cos\left(\frac{2t}{10}\right)\sin 2x}{2^3} + \frac{\cos\left(\frac{3t}{10}\right)\sin 3x}{3^3} - \ldots\right)$$

HEAT

C = 0.1, L = π, f(x) = k(π²x -x³)

$$\lambda_n = \frac{nc\pi}{L} = \frac{n(0.1)\pi}{\pi} = \frac{n}{10}, \quad \lambda_n^2 = \frac{n^2}{100}$$



$$\sum_{n=1}^{\infty} B_n \sin nx\, e^{-\lambda_n^2 t} = 12k\left(\sin x\, e^{-0.01t} - \frac{\sin 2x}{2^3} e^{-0.04t} + \frac{\sin 3x}{3^3} e^{-0.09t} - \ldots\right)$$