NUMERICAL DIFFERENTIATION

3 - POINTS: $(x_{-1},f_{-1}), (x_0,f_0), (x_1,f_1)$ where $h = x_1 - x_0 = x_0 - x_{-1}$

$$f_0' = \frac{f_1 - f_{-1}}{2h} + O(h^2) \qquad f_0'' = \frac{f_1 - 2f_0 + f_{-1}}{h^2} + O(h^2)$$

5 - POINTS: $(x_{-2},f_{-2}),(x_{-1},f_{-1}), (x_0,f_0), (x_1,f_1),(x_2,f_2)$

$$f_0' = \frac{-f_2 + 8f_1 - 8f_{-1} + f_{-2}}{12h} + O(h^4)$$

$$f_0'' = \frac{-f_2 + 16f_1 - 30f_0 + 16f_{-1} + f_{-2}}{12h^2} + O(h^4)$$

$$f_0''' = \frac{f_2 - 2f_1 + 2f_{-1} - f_{-2}}{2h^3} + O(h^4)$$

$$f_0^{(4)} = \frac{f_2 - 4f_1 + 6f_0 - 4f_{-1} + f_{-2}}{h^4} + O(h^4)$$

RICHARDSON'S EXTRAPOLATION:

$$N_1(h) = \frac{f(x_0 + h) - f(x_0 - h)}{2h}, \; N_1\left(\frac{h}{2}\right) = \frac{f\left(x_0 + \frac{h}{2}\right) - f\left(x_0 - \frac{h}{2}\right)}{h}, \; \textit{etc.}$$

$$N_2(h) = \frac{4N_1\left(\frac{h}{2}\right) - N_1(h)}{3}, \; N_2\left(\frac{h}{2}\right) = \frac{4N_1\left(\frac{h}{4}\right) - N_1\left(\frac{h}{2}\right)}{3}, \; \ldots$$