Derive - Extra Credit - Math 417 - Prof. Richard B. Goldstein

(1) Find a power series solution to: $(x + 1)y' - (2x + 3)y = 0, y(0) = 1$

(2) Find the Fourier Series expansion of $f(x) = x^3 - x\pi^2$ on $(-\pi, \pi)$

(3) Find an odd half-range expansion of $f(x) = 2x - x^2$ on (0,1)

(4) Find the Euler and Runge-Kutta numerical solutions to

$$y' = \frac{1}{2}\left(\frac{y}{x} - \frac{x}{y}\right), \; y(2) = 2$$

(5) Find the inverse of the 4 X 4 matrix:
$$\begin{bmatrix} 2 & 0 & 1 & 2 \\ 1 & 1 & 0 & 2 \\ 2 & -1 & 3 & 1 \\ 3 & -1 & 4 & 3 \end{bmatrix}^{-1}$$

(6) Row reduce the matrix:
$$\begin{bmatrix} 1 & -1 & 3 & -3 & 3 \\ -5 & 2 & -5 & 4 & -5 \\ -3 & -4 & 7 & -2 & 7 \\ 2 & 3 & 1 & -11 & 1 \end{bmatrix}$$

(7) Find all three eigenvalues and eigenvectors for the matrix in (5).

(8) Find the **div** and **curl** of the vector: x^2y **i** - y sin xz **j** + $e^{xy}z$ **k**

(9) Find the curvature for **r** = $[t^2 + 2, t^2 - 4t, 2t]$

How to do extra credit using Derive - Prof. Richard B. Goldstein

In all cases, Ctrl + B or Simplify | Basic .. will simplify the expressions if necessary and Simplify | Approximate will show the answers to a given number of digits.

(1) Start by File | Load | Utility... and find ode_appr.mth
Enter expressions by Ctrl + A (Author | Expression) - enter the expression:
taylor_ode1((2x+3)/(x+1),x,y,0,1,6)

(2) Again, File | Load | Utility...and select int_apps.mth
Enter the expression: fourier(x^3-xπ^2,x,-π,π,5)
Note: you will find the π in the symbol table

(3) Enter the expression: chi(-1,x,0)(2x+x^2)+chi(0,x,1)(2x-x^2)
Suppose it expression #k.
Then enter the expression: fourier(#k,x,-1,1,5)

(4) Enter the expression: euler(0.5(y/x-x/y),x,y,2,2,0.2,5)
Enter the expression: rk([0.5(y/x-x/y)],[x,y],[2,2],0.2,5)

(5) Select: Author | Matrix
Suppose the matrix is expression #k
Enter the expression: #k^-1

(6) Select Author | Matrix
Suppose the matrix is expression #k
Enter the expression: row_reduce(#k)

(7) Enter the expression: eigenvalues(#k,x) where #k was the matrix in (5)

Use File | Load | Utilty... and bring in vector.mth
Now, for each eigenvalue enter the expression: approx_eigenvector(#k,λ)
where #k is again the 4 X 4 matrix and each value of λ is entered one at a time.

Note: for the complex numbers **i** (complex $\sqrt{-1}$) is entered as #i
for example, enter 3.4 + 2.8i as 3.4+2.8#i

(8) Enter the expression: v:=[x^2y,-ysin(xz),exp(xy)z]
Enter the expression: div(v)
Enter the expression: curl(v)

(9) Enter the expression: r1:=[t^2+2,t^2-4t,2t]
Enter the expression: curvature3d(r,t):=abs(cross(dif(r,t,1),dif(r,t,2)))/abs(dif(r,t,1))^3
Enter the expression: curvature3d(r1,t)