Unknown But Unequal Variances

Suppose two populations are assumed to have an **approximately normal distribution**. Independent samples from each population are given of size n_1 and n_2 respectively. The sample means and standard deviations are given as $\bar{x}_1, s_1$ and $\bar{x}_2, s_2$ where the hypothesis test:

$$\begin{aligned} H_0&: \sigma_1 = \sigma_2 \\ H_1&: \sigma_1 \neq \sigma_2 \end{aligned}$$

fails to accept H_0. That is, the analyst is **not** able to assume equality of variances. Then the statistic to test:

$$\begin{aligned} H_0&: \mu_1 - \mu_2 = \mu_d \\ H_1&: \mu_1 - \mu_2 > \mu_d \end{aligned}$$

is given by:

$$t' = \frac{(\bar{x}_1 - \bar{x}_2) - \mu_d}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

has an approximate t-distribution with approximately v degrees of freedom where

$$v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$$

EXAMPLE:

Sample	1	2
Size	6	8
Mean	9	5
St.Dev.	2	6

$$F = \frac{s_2^2}{s_1^2} = \frac{36}{4} = 9.0 > 6.85 = F_{0.025,7,5}$$

Using $\mu_d = 0,\ t' = \dfrac{9 - 5}{\sqrt{\frac{4}{6} + \frac{36}{8}}} \approx 176$

$$v = \frac{\left(\frac{4}{6} + \frac{36}{8}\right)^2}{\frac{\left(\frac{4}{6}\right)^2}{6 - 1} + \frac{\left(\frac{36}{8}\right)^2}{8 - 1}} = 8.953$$

the null hypothesis would fail to be rejected (accepted) at the 5% level.