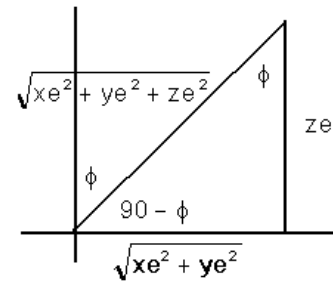
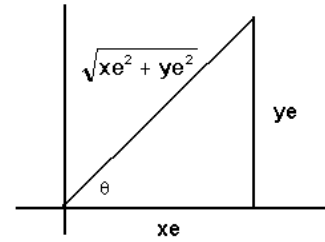
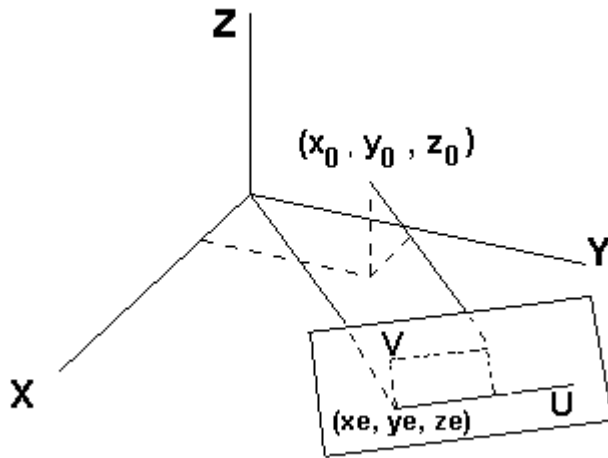


## 2-D Rendering of 3-D Objects Projection and Perspective

*Prof. Richard B. Goldstein*



To go from a 3-D coordinate system of  $(x, y, z)$  to a 2-D coordinate system of  $(u, v)$  two rotations are made. The first is a rotation of angle  $90^\circ + \theta$  from the  $x$ -axis and the second is a rotation of  $-\phi$  from the  $z$ -axis. The new coordinates are found by multiplying two matrices:

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} -\sin \theta & \cos \theta & 0 \\ -\cos \theta & -\sin \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\sin \theta & \cos \theta & 0 \\ -\cos \theta \cos \phi & -\cos \theta \sin \phi & -\sin \phi \\ -\sin \theta \cos \phi & -\sin \theta \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Since

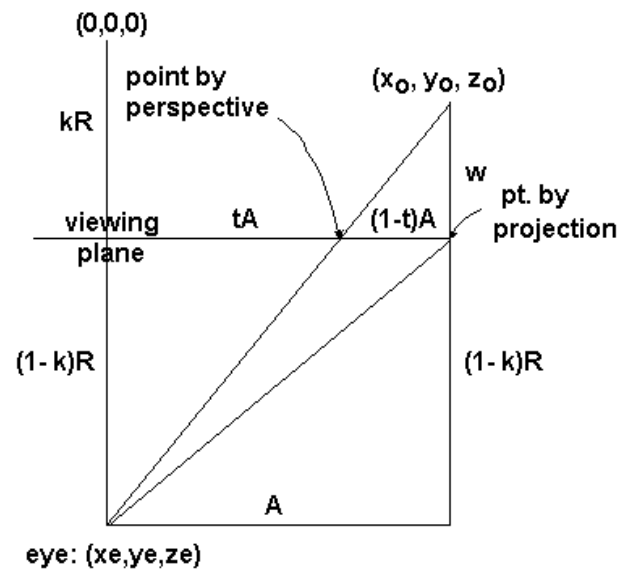
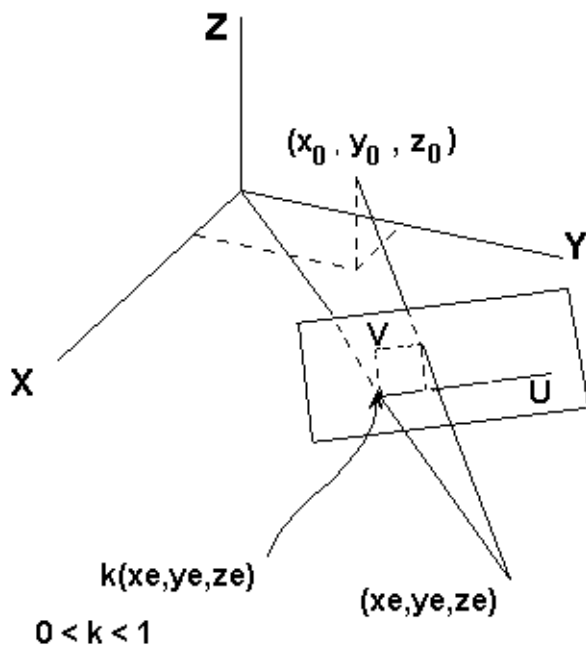
$$\cos \theta = \frac{xe}{\sqrt{xe^2 + ye^2}}, \sin \theta = \frac{ye}{\sqrt{xe^2 + ye^2}}, \cos \phi = \frac{ze}{\sqrt{xe^2 + ye^2 + ze^2}}, \text{ and } \sin \phi = \frac{\sqrt{xe^2 + ye^2}}{\sqrt{xe^2 + ye^2 + ze^2}}$$

$$u = -x_0 \sin \theta + y_0 \cos \theta = \frac{-x_0 * ye + y_0 * xe}{\sqrt{xe^2 + ye^2}}$$

$$v = -x_0 \cos \theta \cos \phi - y_0 \sin \theta \cos \phi + z_0 \sin \phi = \frac{-(x_0 * xe + y_0 * ye) * ze + z_0 (xe^2 + ye^2)}{\sqrt{xe^2 + ye^2} \sqrt{xe^2 + ye^2 + ze^2}}$$

Using screen graphics with the origin at  $(cx, cy)$  and  $scale=s$  the new point is located at  $(cx + s*u, cy - s*v)$ . Remember, the  $y$ -axis is upside down in screen graphics.

If the picture is in perspective, then use the following



Using the common ratios in the triangle shown on the right the  $(u,v)$  coordinates are reduced by a factor of  $t$ . The distance from the point  $(x_0, y_0, z_0)$  to the viewing plane is  $w$  (assume  $w > 0$ ).

$$\frac{(1-t)A}{w} = \frac{A}{(1-k)R + w}$$

where  $R = \sqrt{x_e^2 + y_e^2 + z_e^2}$  and  $w = \frac{k(R^2) - x_0 * x_e - y_0 * y_e - z_0 * z_e}{R}$

$$t = 1 - \frac{w}{(1-k)R + w} = \frac{1-k}{1 - \left( \frac{x_0 * x_e + y_0 * y_e + z_0 * z_e}{x_e^2 + y_e^2 + z_e^2} \right)}$$

which gives the new  $u$  and  $v$  coordinates as:

$$u = t * \left( \frac{y_0 * x_e - x_0 * y_e}{\sqrt{x_e^2 + y_e^2}} \right)$$

$$v = t * \left( \frac{(x_e^2 + y_e^2)z_0 - (x_0 * x_e + y_0 * y_e) * z_e}{R\sqrt{x_e^2 + y_e^2}} \right)$$

Note:  $k$  only affects the scaled size of objects, not their shape.