Math 107. Prof. Richard B. Goldstein – Barnett (2nd Ed) – Chap 8 HW

Sect 8.1

#7 Die 3 sides red, 2 white, 1 blue $P\{\text{Blue} \cup \text{White}\} = \frac{1+2}{6} = \frac{3}{6} = \boxed{\frac{1}{2}}$

#15 $P\{\text{black jack}\} = \frac{2}{52} = \boxed{\frac{1}{26}}$ (Jack of clubs, spades)

#23 (A) $P(J) = 0.15$, $P(G) = -0.35$ ... not allowed

(B) $P(J) = 0.32$, $P(G) = 0.28$, $P(P) = 0.24$, $P(S) = 0.30$ Sum = 1.14 ≠ 0
not allowed

(C) $P(J) = 0.26$, $P(G) = 0.14$, $P(P) = 0.30$, $P(S) = 0.30$ Sum = 1 ✓
no value below 0 ✓

#27 3 children
$S = \{GGG, GGB, GBG, GBB, BGG, BGB, BBG, BBB\}$ all equally likely $\frac{1}{8}$
any order $P\{GGB, GBG, BGG\} = \frac{3}{8}$, but particular order $\boxed{P(GGB) = \frac{1}{8}}$

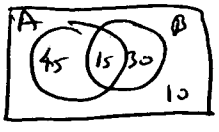
#37 5 thank you notes, 5 envelopes
$5! = 120$ arrangements, only one correct $\boxed{P = \frac{1}{120}}$

#51 $P\{\text{dice sum} = 7 \text{ or } 11\} = \frac{6+2}{36} = \frac{8}{36} = \boxed{\frac{2}{9}}$

#53 $P\{\text{dice sum divisible by 2 or 3}\} = P\{2 \text{ or } 3 \text{ or } 4 \text{ or } 6 \text{ or } 8 \text{ or } 9 \text{ or } 10 \text{ or } 12\}$
$= P(2) + P(3) + P(4) + P(6) + P(8) + P(9) + P(10) + P(12)$
$= \frac{1+2+3+5+5+4+3+1}{36} = \frac{24}{36} = \boxed{\frac{2}{3}}$

Sect 8.2

#3 $P(A' \cup B)$



$= \frac{15+30+10}{100} = \boxed{0.55}$

#11 $P(D \cap F) = P(\text{diamond} \cap \text{face card}) = \boxed{\frac{3}{52}}$ J, Q, K of ♢

#17 $P(D \cup F') = P(D) + P(F') - P(D \cap F') = \frac{13}{52} + \frac{40}{52} - \frac{10}{52} = \boxed{\frac{43}{52}}$ (all but 9 cards)

#21 $P(\text{divisible by 3 or 4}) = P(3) + P(4) + P(6) + P(8) + P(9) + P(12) + P(15) + P(16) + P(18) + P(20) + P(21) + P(24) = \boxed{\frac{12}{25} = 0.48}$
$S = \{1, 2, \ldots, 25\}$

Sect 8.2

#33 (A) odds $\frac{3}{8}$ $\quad P(E) = \frac{3}{3+8} = \frac{3}{11}$

(B) odds $\frac{11}{7}$ $\quad P(E) = \frac{11}{11+7} = \frac{11}{18}$

(C) odds $\frac{4}{1}$ $\quad P(E) = \frac{4}{4+1} = \frac{4}{5}$

(D) odds $\frac{49}{51}$ $\quad P(E) = \frac{49}{49+51} = \frac{49}{100}$

#55 $P(\underset{\text{card}}{\text{black}} \text{ or } \text{ace}) = P(\underset{\text{card}}{\text{black}}) + P(\text{ace}) - P(\underset{\text{of ace}}{\text{black card}})$

$= \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52} = \boxed{\frac{7}{13}}$

odds $\frac{7}{13}$ to $1 - \frac{7}{13} = \frac{6}{13}$ or 7 to 6 or $\boxed{\frac{7}{6}}$

Sect 8.3

#9 $P(D|C) = \frac{P(D \cap C)}{P(C)} = \frac{0.06}{0.20} = \boxed{0.3}$ from table (p 429 text)

#17 E and D indep?

$P(D) = 0.20 \neq P(D|C)$ $\therefore$ dependent

#23

e_i	1	2	3	4	5
$P(e_i)$	0.3	0.1	0.2	0.3	0.1

E = even no. 2, 4
F = number less than 4 1, 2, 3

(A) $P(F|E) = \frac{P(F \cap E)}{P(E)} = \frac{P(2)}{P(2)+P(4)} = \frac{0.1}{0.1+0.3} = \frac{0.1}{0.4} = \boxed{\frac{1}{4}}$

(B) $P(F) = P(1)+P(2)+P(3) = 0.3+0.1+0.2 = 0.6 \neq P(F|E)$ $\therefore$ dependent

#27 $S = \{HH, HT, TH, TT\}$

E_1 = head on 1st toss, E_2 = tail on 1st toss, E_3 = tail on 2nd toss, E_4 = head on 2nd toss

(A) E_1 and E_4 $\quad P(E_1) = P\{HH, HT\} = \frac{1}{2}$ $\quad P(E_4) = P(HH, TH) = \frac{1}{2}$ $\quad \frac{1}{4} = \frac{1}{2} \cdot \frac{1}{2}$ ✓

$P(E_1 \cap E_4) = P(HH) = \frac{1}{4}$ indep

(B) E_1 and E_2 $\quad P(E_1) = \frac{1}{2}$ $\quad P(E_2) = P\{TH, TT\} = \frac{1}{2}$ $\quad P(E_1 \cap E_2) = P(\emptyset) = 0$ not mutually excl

$\frac{1}{2} \cdot \frac{1}{2} \neq 0$ $\therefore$ dep mutually excl

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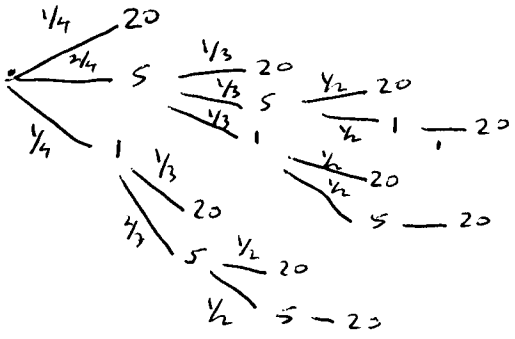
Sect 8.3

#31 (A) $P(1^{st}\text{ club}, 2^{nd}\text{ heart}) = \frac{13}{52}\cdot\frac{13}{51} = \frac{13}{204} = 0.0637...$ ← W/o repl.

(B) $P(1^{st}\text{ club}, 2^{nd}\text{ heart}) = \frac{13}{52}\cdot\frac{13}{52} = \frac{1}{16} = 0.0625$ ← W repl.

#61 1 \$20 2 \$5 1 \$1 W/o repl. until a \$20 is withdrawn



(A) \$26,000 bonus

(5, 1, 20) $P = \frac{2}{4}\cdot\frac{1}{3}\cdot\frac{1}{2} = \frac{2}{24}$

(1, 5, 20) $P = \frac{1}{4}\cdot\frac{2}{3}\cdot\frac{1}{2} = \frac{2}{24}$

$\frac{4}{24} = \boxed{\frac{1}{6}}$

(B) \$31,000

(5, 5, 1, 20)
(5, 1, 5, 20)
(1, 5, 5, 20)

each $P = \frac{2}{4}\cdot\frac{1}{3}\cdot\frac{1}{2}\cdot 1 = \frac{2}{24}$

$3\times\frac{2}{24} = \frac{6}{24} = \boxed{\frac{1}{4}}$

(C) $P\{\text{ends at 3rd draw}\}$

(5, 5, 20) $\frac{2}{4}\cdot\frac{1}{3}\cdot\frac{1}{2} = \frac{2}{24}$
(5, 1, 20) $\frac{2}{4}\cdot\frac{1}{3}\cdot\frac{1}{2} = \frac{2}{24}$
(1, 5, 20) $\frac{1}{24}\cdot\frac{2}{3}\cdot\frac{1}{2} = \frac{2}{24}$

Sum $= \frac{6}{24} = \boxed{\frac{1}{4}}$

Sect 8.4

#1 $P(M\cap A) = P(M)\,P(A|M) = 0.6(0.8) = \boxed{0.48}$

#3 $P(A) = P(M\cap A) + P(M'\cap A) = 0.6(0.8) + 0.4(0.3) = 0.48 + 0.12 = \boxed{0.60}$

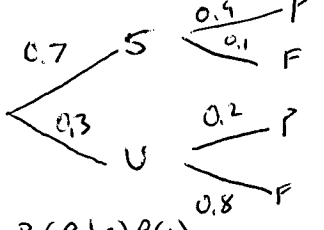
#5 $P(M|A) = \frac{0.48}{0.48+0.12} = \frac{0.48}{0.60} = \boxed{0.80}$ based upon P437 tree

#11 $P(U|C) = \frac{P(U\cap C)}{P(C)} = \frac{P(U)P(C|U)}{P(U)P(C|U)+P(V)P(C|V)+P(W)P(C|W)} = \frac{0.2(0.4)}{0.2(0.4)+0.5(0.2)+0.3(0.6)}$

$= \frac{0.08}{0.08+0.10+0.18} = \frac{0.08}{0.36} = \boxed{\frac{2}{9}} = 0.222...$

#47 S = Satisfactory
U = Unsat.
P = Pass
F = fail

$P(U) = 0.3$
$P(S) = 0.7$



(A) $P(S|P) = \frac{P(S\cap P)}{P(P)} = \frac{P(P|S)P(S)}{P(P|S)P(S)+P(P|U)P(U)} = \frac{0.7(0.9)}{0.7(0.9)+0.3(0.2)} = \frac{0.63}{0.63+0.06}$

$= \frac{63}{69} = \boxed{0.913}$

(B) $P(S|F) = \frac{P(S\cap F)}{P(F)} = \frac{P(S)P(F|S)}{P(S)P(F|S)+P(U)P(F|U)} = \frac{0.7(0.1)}{0.7(0.1)+0.3(0.8)} = \frac{7}{31} = \boxed{0.226}$

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Sect 8.4

	prob	cond. prob	joint prob	revised probs
#49	$P(A) = 0.2$	$P(\text{def}\mid A) = 0.01$	$P(A \cap \text{def}) = 0.002$	$P(A\mid \text{def}) = 2/22 = 0.091$
	$P(B) = 0.4$ X	$P(\text{def}\mid B) = 0.03$ =	$P(B \cap \text{def}) = 0.012$	$P(B\mid \text{def}) = 12/22 = 0.545$
	$P(C) = 0.4$	$P(\text{def}\mid C) = 0.02$	$P(C \cap \text{def}) = 0.008$	$P(C\mid \text{def}) = 8/22 = 0.364$
	1.0		0.022	1.00

answers

Sect 8.5

#1 $EX = (-3)(0.3) + 0(0.5) + 4(0.2) = -0.9 + 0 + 0.8 = \boxed{-0.1}$

#7

X	1	2	3	4	5	6
P	1/6	1/6	1/6	1/6	1/6	1/6
net	−3	−2	−1	0	+1	+2

$EX = (-3)\frac{1}{6} + (-2)\frac{1}{6} + (-1)\frac{1}{6} + 0\left(\frac{1}{6}\right) + 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right)$

$= -\frac{3}{6} = \boxed{-\$0.50}$ Game is not fair

#15 Sum = 7 −$10
Sum = 11, 12 +$11
Sum = otherwise X

$E = 0 = -10\left(\frac{6}{36}\right) + 11\left(\frac{2+1}{36}\right) + X\left(\frac{27}{36}\right)$

$-\frac{60}{36} + \frac{33}{36} + \frac{27X}{36} = 0$ $\quad -27 + 27X = 0$

$\boxed{X = \$1}$

#23 $E = 35\left(\frac{1}{38}\right) + (-1)\left(\frac{37}{38}\right) = \frac{35}{38} - \frac{37}{38} = \frac{-2}{38} = -0.0526\ldots = \boxed{-5.26¢}$

#27 tickets $1
$EX = 500\left(\frac{1}{5000}\right) + 100\left(\frac{3}{5000}\right) + 20\left(\frac{5}{5000}\right) + 5\left(\frac{20}{5000}\right) = \frac{1000}{5000} = \boxed{\$0.20}$ on ticket

$\therefore -1 + 0.2 = \boxed{-\$0.80}$ net

#37 premium = $150

insur payment $EX = 5000(0.01) + 0(0.99) = 50$

} expected return $= 50 - 150 = \boxed{-\$100}$

#39

	site		
A	+30 M	prob = 0.2	
	−3 M	prob = 0.8	
	$E(A) = 30(0.2) + (-3)(0.8) = \boxed{\$3.6\text{ M}}$		
B	+70 M	prob = 0.1	
	−4 M	prob = 0.9	
	$E(B) = 70(0.1) + (-4)(0.9)$		
	$= 7 - 3.6 = 3.4$		

(A) higher expected value