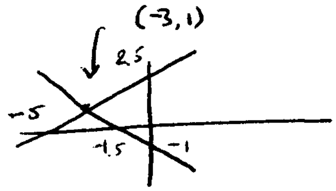
Math 107 - Prof. Richard B. Goldstein - Chap 4 HW - Barnett 2nd ed

Sect 4.1

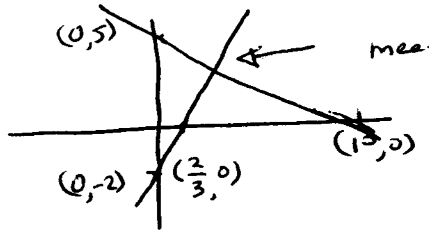
#3 $-x+2y=5$
$2x+3y=-3$



meet at $(-3,1)=(x,y)$

A

#5 $3x-y=2$
$x+2y=10$



meet at $(2,4)$ $\boxed{x=2,\ y=4}$

#11 $2x+y=6$
$x-y=-3$

$\Rightarrow y=6-2x$

$x-(6-2x)=-3 \Rightarrow 3x-6=-3$
$3x=3$
$\boxed{x=1}$

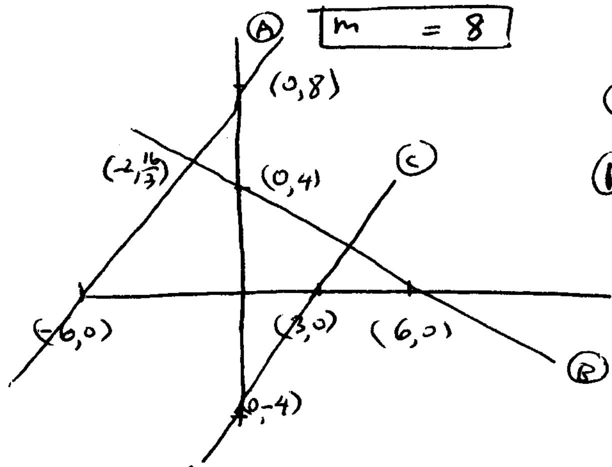
$y=6-2(1)=4$
$\boxed{y=4}$

#15 $2m-n=10$
$m-2n=-4$

$\times(-2)$ $-4m+2n=-20$
$m-2n=-4$
$-3m=-24$
$\boxed{m=8}$

$2(8)-n=10$
$16-n=10$
$-n=-6$
$\boxed{n=6}$

#47 $4x-3y=-24$
$2x+3y=12$
$8x-6y=24$



A + B $\left(-2, \frac{16}{3}\right)$

B + C $\left(4, \frac{4}{3}\right)$

A // C

#53

price	supply (s)	demand (d)
(p) \$4.80 /bushel	1.9 billion	2.0 billion
5.10	2.1	1.8

Supply $m=\frac{2.1-1.9}{5.1-4.8}=\frac{0.2}{0.3}=\frac{2}{3}$

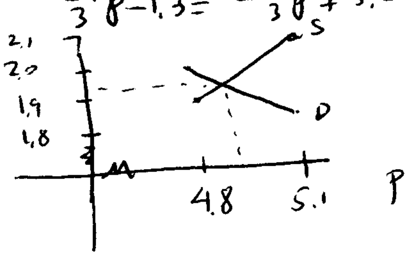
$s-1.9=\frac{2}{3}(p-4.80) \Rightarrow \boxed{s=\frac{2}{3}p-1.3}$

demand $m=\frac{1.8-2.0}{5.1-4.8}=\frac{-0.2}{0.3}=-\frac{2}{3}$

$d-2.0=-\frac{2}{3}(p-4.80) \Rightarrow \boxed{d=-\frac{2}{3}p+5.2}$

$s=d$ $\frac{2}{3}p-1.3=-\frac{2}{3}p+5.2$

$\frac{4}{3}p=6.5$ $\quad p=4.875$ $\quad s=d=$ 1.95 billion



Sect 4.2

#7 $A = \begin{bmatrix} 2 & -4 & 0 \\ 6 & 1 & -5 \end{bmatrix}$ $\boxed{a_{12} = -4}$ $\boxed{a_{23} = -5}$

#15 $-4R_1 \to R_1$ $\left[\begin{array}{cc|c} 1 & -3 & 2 \\ 4 & -6 & -8 \end{array}\right] \Rightarrow \boxed{\left[\begin{array}{cc|c} -4 & 12 & -8 \\ 4 & -6 & -8 \end{array}\right]}$

#21 $(-2)R_1 + R_2 \to R_2$ $(-2)\left[\begin{array}{cc|c} 1 & -3 & 2 \end{array}\right] + \left[\begin{array}{cc|c} 4 & -6 & -8 \end{array}\right]$

$\left[\begin{array}{cc|c} -2 & 6 & -4 \end{array}\right] + \left[\begin{array}{cc|c} 4 & -6 & -8 \end{array}\right] = \boxed{\left[\begin{array}{cc|c} 2 & 0 & -12 \end{array}\right]}$

#45 $3x_1 - x_2 = 2$
$x_1 + 2x_2 = 10$ $\left[\begin{array}{cc|c} 3 & -1 & 2 \\ 1 & 2 & 10 \end{array}\right]$ switch $R_1 \leftrightarrow R_2$ $\left[\begin{array}{cc|c} 1 & 2 & 10 \\ 3 & -1 & 2 \end{array}\right]$

$(-3)R_1 + R_2 \to R_2$ $\left[\begin{array}{cc|c} 1 & 2 & 10 \\ 0 & -7 & -28 \end{array}\right]$ $(-\frac{1}{7})R_2 \to R_2$ $\left[\begin{array}{cc|c} 1 & 2 & 10 \\ 0 & 1 & 4 \end{array}\right]$

$(-2)R_2 + R_1 \to R_1$ $\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 4 \end{array}\right]$ $\therefore$ $\boxed{x_1 = 2 \quad x_2 = 4}$

#59 $-4x_1 + 6x_2 = -8$
$6x_1 - 9x_2 = 12$ $\left[\begin{array}{cc|c} -4 & 6 & -8 \\ 6 & -9 & 12 \end{array}\right]$ $(\frac{3}{2})R_1 + R_2 \to R_2$ $\left[\begin{array}{cc|c} -4 & 6 & -8 \\ 0 & 0 & 0 \end{array}\right]$

$\therefore$ two coincident lines (∞ solutions) infinite

let $\boxed{x_1 = t}$ then $-4t + 6x_2 = -8$
$6x_2 = 4t - 8$
$\boxed{x_2 = \frac{2}{3}t - \frac{4}{3}}$

Sect 4.3

#9 $\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right]$ is not in row-reduced form $\boxed{2R_2 + R_1 \to R_1}$

#17 $\left[\begin{array}{ccc|c} 1 & 0 & -3 & 5 \\ 0 & 1 & 2 & -7 \end{array}\right]$ $x_1 \quad -3x_3 = 5$
$x_2 + 2x_3 = -7$ $x_3 = t$ $x_1 - 3t = 5$
$x_2 + 2t = -7$
$\boxed{x_1 = 5 + 3t}$
$\boxed{x_2 = -7 - 2t}$
$\boxed{x_3 = t \text{ arb}}$

#29 $\left[\begin{array}{ccc|c} 1 & 2 & -2 & -1 \\ 0 & 3 & -6 & 1 \\ 0 & -1 & 2 & -\frac{1}{3} \end{array}\right] \xrightarrow[(\frac{1}{3})R_2 \to R_2]{} \left[\begin{array}{ccc|c} 1 & 2 & -2 & -1 \\ 0 & 1 & -2 & 1/3 \\ 0 & -1 & 2 & -1/3 \end{array}\right] \xrightarrow[R_2 + R_3 \to R_3]{(-2)R_2 + R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & 0 & 2 & -5/3 \\ 0 & 1 & -2 & 1/3 \\ 0 & 0 & 0 & 0 \end{array}\right]$

now row reduced form let $\boxed{x_3 = t \text{ arb.}}$ $x_1 + 2t = -5/3$
$x_2 - 2t = \frac{1}{3}$
$\boxed{x_1 = -\frac{5}{3} - 2t}$
$\boxed{x_2 = \frac{1}{3} + 2t}$

#31 $2x_1 + 4x_2 - 10x_3 = -2$
$3x_1 + 9x_2 - 21x_3 = 0$
$x_1 + 5x_2 - 12x_3 = 1$

$\left[\begin{array}{ccc|c} 1 & 5 & -12 & 1 \\ 3 & 9 & -21 & 0 \\ 2 & 4 & -10 & -2 \end{array}\right] \xrightarrow[(-2)R_1 + R_3 \to R_3]{(-3)R_1 + R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 5 & -12 & 1 \\ 0 & -6 & 15 & -3 \\ 0 & -6 & 14 & -4 \end{array}\right]$ $(-\frac{1}{6})R_2 \to R_2$ $\left[\begin{array}{ccc|c} 1 & 5 & -12 & 1 \\ 0 & 1 & -5/2 & 1/2 \\ 0 & -6 & 14 & -4 \end{array}\right]$

$-5R_2 + R_1 \to R_1$
$6R_2 + R_3 \to R_3$ $\left[\begin{array}{ccc|c} 1 & 0 & 1/2 & -3/2 \\ 0 & 1 & -5/2 & 1/2 \\ 0 & 0 & -1 & -1 \end{array}\right]$ $-R_3 \to R_3$ $\left[\begin{array}{ccc|c} 1 & 0 & 1/2 & -3/2 \\ 0 & 1 & -5/2 & 1/2 \\ 0 & 0 & 1 & 1 \end{array}\right]$ $\xrightarrow[\frac{5}{2}R_3 + R_2 \to R_2]{-\frac{1}{2}R_3 + R_1 \to R_1}$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{array}\right]$ $\boxed{\begin{array}{l} x_1 = -2 \\ x_2 = 3 \\ x_3 = 1 \end{array}}$

Sect 4.3

#37

$$\begin{aligned} 2x_1 - x_2 &= 0 \\ 3x_1 + 2x_2 &= 7 \\ x_1 - x_2 &= -1 \end{aligned}$$

$$\left[\begin{array}{cc|c} 1 & -1 & -1 \\ 3 & 2 & 7 \\ 2 & -1 & 0 \end{array}\right] \begin{array}{l} -3R_1 + R_2 \to R_2 \\ -2R_1 + R_3 \to R_3 \end{array}$$

$$\left[\begin{array}{cc|c} 1 & -1 & -1 \\ 0 & 5 & 10 \\ 0 & 1 & 2 \end{array}\right] \quad R_2 \leftrightarrow R_3 \quad \left[\begin{array}{cc|c} 1 & -1 & -1 \\ 0 & 1 & 2 \\ 0 & 5 & 10 \end{array}\right] \begin{array}{l} R_2 + R_1 \to R_1 \\ -5R_2 + R_3 \to R_3 \end{array} \quad \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right] \quad \boxed{x_1 = 1,\ x_2 = 2}$$

Sect 4.4

#5

$$\begin{bmatrix} 4 & -5 \\ 1 & 0 \\ 1 & -3 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 6 & -2 \\ 1 & -7 \end{bmatrix} = \begin{bmatrix} 5^{*} & -7 \\ -5 & 2 \\ 0 & 4 \end{bmatrix} \qquad * \ 4-(-1) = 5 \text{ etc}$$

#11

$$\begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} (2)(1)+(-3)(0) & (2)(-1)+(-3)(-2) \\ (1)(1)+(2)(0) & (1)(-1)+(2)(-2) \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 1 & -5 \end{bmatrix}$$

#19

$$\begin{bmatrix} 5 & -2 \end{bmatrix}\begin{bmatrix} -3 \\ -4 \end{bmatrix} = (5)(-3) + (-2)(-4) = -15 + 8 = \boxed{[-7]}$$

#37

$$(3)BA + (4)AC = 3\underbrace{\underset{2\times 2}{\begin{bmatrix} -3 & 1 \\ 2 & 5 \end{bmatrix}}\underset{2\times 3}{\begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & -2 \end{bmatrix}}}_{2\times 3} + 4\underbrace{\underset{2\times 3}{\begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & -2 \end{bmatrix}}\underset{3\times 3}{\begin{bmatrix} -1 & 0 & 2 \\ 4 & -3 & 1 \\ -2 & 3 & 5 \end{bmatrix}}}_{2\times 3 \text{ ok}}$$

$$= 3\begin{bmatrix} -6 & 7 & -11 \\ 4 & 18 & -4 \end{bmatrix} + 4\begin{bmatrix} -12 & 12 & 18 \\ 20 & -18 & -6 \end{bmatrix} = \boxed{\begin{bmatrix} -66 & 69 & 39 \\ 92 & -18 & -36 \end{bmatrix}}$$

#53

$$\begin{bmatrix} 2x & 4 \\ -3 & 5x \end{bmatrix} + \begin{bmatrix} 3y & -2 \\ -2 & -y \end{bmatrix} = \begin{bmatrix} -5 & 2 \\ -5 & 13 \end{bmatrix}$$

$$\begin{aligned} 2x + 3y &= -5 \\ 5x - y &= 13 \quad (\times 3) \\ 15x - 3y &= 39 \\ \hline 17x &= 34 \end{aligned} \qquad x = 2$$

$$4 + 3y = -5 \qquad y = -3$$

$4 - 2 = 2$ ✓

$-3 - 2 = -5$ ✓

#61

$$\frac{1}{2}\left(\begin{bmatrix} 47 & 39 \\ 90 & 125 \end{bmatrix} + \begin{bmatrix} 56 & 42 \\ 84 & 115 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 103 & 81 \\ 174 & 240 \end{bmatrix} = \begin{array}{cc} \text{Guitar} & \text{Banjo} \\ \left[\begin{array}{cc} 51.50 & 40.50 \\ 87.00 & 120.00 \end{array}\right] & \begin{array}{l} \text{Material} \\ \text{Labor} \end{array} \end{array}$$

#65

$$MN \quad \begin{array}{ccc} \text{Cut} & \text{Assemb.} & \text{Pack} \\ \hline \end{array}$$

$$\begin{bmatrix} 0.6 & 0.6 & 0.2 \\ 1.0 & 0.9 & 0.3 \\ 1.5 & 1.2 & 0.4 \end{bmatrix} \begin{bmatrix} 17.30 & 14.65 \\ 12.22 & 10.29 \\ 10.63 & 9.66 \end{bmatrix} \begin{array}{l} \text{Cut} \\ \text{Assemb} \\ \text{Pack} \end{array} = \begin{array}{c} \\ (1) \\ (2) \\ (4) \end{array} \begin{array}{cc} \text{MA} & \text{VA} \\ \left[\begin{array}{cc} 19.84 & 16.90 \\ 31.49 & 26.81 \\ 44.87 & 38.19 \end{array}\right] \end{array}$$

Sect 4.5

#9 $\begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} (3)(3)+(-4)(2) & (3)(4)+(-4)(3) \\ (-2)(3)+(3)(2) & (-2)(4)+(3)(3) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

INVERSE!

#11 $\begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 2-2 & 2-2 \\ -1+1 & -1+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ not an inverse

#19 $\begin{bmatrix} 1 & 2 & 0 \\ -3 & 2 & -1 \end{bmatrix}$ 2×3 not square ∴ no inverse

#23 $\begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}$ no inverse - row of 0's or determinant of 0

#41 $\begin{bmatrix} 2 & 6 \\ 3 & 9 \end{bmatrix}^{-1}$ does not exist $D = (2)(9) - (6)(3) = 18 - 18 = 0$

#47 $\left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ -1 & -1 & 0 & 0 & 0 & 1 \end{array}\right] R_1 \leftrightarrow R_2 \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & 1 & 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} -2R_1 + R_2 \to R_2 \\ R_1 + R_3 \to R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right]$

no inverse

#65

T E

$\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 20 & 5 & 19 & 14 & 1 & 19 & 0 & 9 & 5 \\ 8 & 0 & 21 & 0 & 12 & 15 & 18 & 19 & 19 \end{bmatrix}$

H Blank

$= \begin{bmatrix} 36 & 5 & 61 & 14 & 25 & 49 & 36 & 47 & 43 \\ 44 & 5 & 82 & 14 & 37 & 64 & 54 & 66 & 62 \end{bmatrix} \ldots$

36 44 5 5 . . . 43 62

Sect 4.6

#7 $x_1 - 3x_2 + 2x_3 = -3$
$-2x_1 + 3x_2 = 1$
$x_1 + x_2 + 4x_3 = -2$

$$\boxed{\begin{bmatrix} 1 & -3 & 2 \\ -2 & 3 & 0 \\ 1 & 1 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}}$$

#15 $\begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 10 \end{bmatrix}$

$x_1 + x_2 = 15$ (×3) $\quad 3x_1 + 3x_2 = 45$
$2x_1 - 3x_2 = 10 \quad\quad 2x_1 - 3x_2 = 10$
$5x_1 = 55$
$x_1 = 11$

$11 + x_2 = 15$
$x_2 = 4$

#17 $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 9 \\ 9 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$

(−) $x_1 + 2x_2 = 6$
$x_1 + x_2 = 5$
$x_2 = 1$

$x_1 + 1 = 5$
$x_1 = 4$

Math 107 (5)
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Sect 4.6

#25

$$x_1 + 3x_2 = k_1 \qquad 2x_1 + 7x_2 = k_2$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}^{-1} = \frac{\begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}}{(1)(7)-(3)(2)} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

for (A) $k_1 = 2$, $k_2 = -1$ $\quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 17 \\ -5 \end{bmatrix}$ $\quad x_1 = 17 \quad x_2 = -5$

(B) $k_1 = 1$, $k_2 = 0$ $\quad \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \end{bmatrix}$ $\quad x_1 = 7 \quad x_2 = -2$

(C) $k_1 = 3$, $k_2 = -1$ $\quad \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 24 \\ -7 \end{bmatrix}$ $\quad x_1 = 24 \quad x_2 = -7$

#41

$$AX - C = D - BX$$
$$AX + BX = C + D$$
$$(A+B)X = C + D \quad \Longrightarrow \quad \boxed{X = (A+B)^{-1}(C+D)}$$

Sect 4.7

$$M = \begin{array}{c|cc} & A & E \\ \hline A & 0.4 & 0.2 \\ E & 0.2 & 0.1 \end{array}$$

#1 40¢ from A, 20¢ from E

#2 20¢ from A, 10¢ from E

#3

$$I - M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.4 & 0.2 \\ 0.2 & 0.1 \end{bmatrix} = \begin{bmatrix} 0.6 & -0.2 \\ -0.2 & 0.9 \end{bmatrix}$$

$$(I-M)^{-1} = \frac{\begin{bmatrix} 0.9 & 0.2 \\ 0.2 & 0.6 \end{bmatrix}}{(0.6)(0.9) - (-0.2)(-0.2) = 0.5} = \boxed{\begin{bmatrix} 1.8 & 0.4 \\ 0.4 & 1.2 \end{bmatrix}}$$

#4

$$(I-M)^{-1}D_1 = \begin{bmatrix} 1.8 & 0.4 \\ 0.4 & 1.2 \end{bmatrix}\begin{bmatrix} 6 \\ 4 \end{bmatrix} = \boxed{\begin{bmatrix} 12.4 \\ 7.2 \end{bmatrix} = \begin{bmatrix} A \\ E \end{bmatrix}}$$

#5

$$(I-M)^{-1}D_2 = \begin{bmatrix} 1.8 & 0.4 \\ 0.4 & 1.2 \end{bmatrix}\begin{bmatrix} 8 \\ 5 \end{bmatrix} = \boxed{\begin{bmatrix} 16.4 \\ 9.2 \end{bmatrix} = \begin{bmatrix} A \\ E \end{bmatrix}}$$

#15

$$M = \begin{bmatrix} 0.7 & 0.8 \\ 0.3 & 0.2 \end{bmatrix} \qquad I - M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.7 & 0.8 \\ 0.3 & 0.2 \end{bmatrix} = \begin{bmatrix} 0.3 & -0.8 \\ -0.3 & 0.8 \end{bmatrix}$$

$$|I - M| = 0 = (0.3)(0.8) - (-0.8)(-0.3) \leftarrow$$ inverse does not exist