Math 132 - Prof. Richard B. Goldstein - Chap 9 HW - Stewart 6th ed

Sect 9.1

#1 $y = x - x^{-1}$, $y' = 1 + x^{-2}$ $\Big\} \Rightarrow xy' + y = x(1+x^{-2}) + (x - x^{-1}) = x + x^{-1} + x - x^{-1} = 2x$ ✓

#5 $y'' + y = \sin x$

(a) $y = \sin x \Rightarrow y' = \cos x,\ y'' = -\sin x \Rightarrow y'' + y = -\sin x + \sin x = 0$ $\boxed{\text{no}}$

(b) $y = \cos x \Rightarrow y' = -\sin x,\ y'' = -\cos x \Rightarrow y'' + y = -\cos x + \cos x = 0$ $\boxed{\text{no}}$

(c) $y = \frac{1}{2}x\sin x \Rightarrow$ $y' = \frac{1}{2}\sin x + \frac{1}{2}x\cos x$, $y'' = \frac{1}{2}\cos x + \frac{1}{2}\cos x - \frac{1}{2}x\sin x = \cos x - \frac{1}{2}x\sin x$ $\Big\} \Rightarrow y'' + y = \cos x - \frac{1}{2}x\sin x + \frac{1}{2}x\sin x = \cos x$ $\boxed{\text{no}}$

(d) $y = -\frac{1}{2}x\cos x \Rightarrow$ $y' = -\frac{1}{2}\cos x + \frac{1}{2}x\sin x$, $y'' = +\frac{1}{2}\sin x + \frac{1}{2}\sin x + \frac{1}{2}x\cos x = \sin x + \frac{1}{2}x\cos x$ $\Big\} \Rightarrow y'' + y = \sin x + \frac{1}{2}x\cos x - \frac{1}{2}x\cos x = \sin x$ $\boxed{\text{yes}}$

#7 $y' = -y^2$ decreasing function

$y = \frac{1}{x+c} = (x+c)^{-1}$, $y' = -(x+c)^{-2}$ $\Big\}$ $-y^2 = -\left[(x+c)^{-1}\right]^2 = -(x+c)^{-2} = y'$ ✓

$\frac{dy}{dx} = -y^2 \Rightarrow -\frac{dy}{y^2} = dx \Rightarrow \frac{1}{y} = x + c$ (Separation method)

but $y = 0$ would work too

$y(0) = 0.5$ $\quad y(0) = \frac{1}{0+c} = 0.5 \Rightarrow c = 2 \quad \therefore \boxed{y = \frac{1}{x+2}}$

#9 $\frac{dP}{dt} = 1.2\,P\left(1 - \frac{P}{4200}\right)$ $\quad$ P is increasing for $0 < P < 4200$ (by both factors being positive)

P is decreasing if $P > 4200$

P is at equilibrium if $P = 0$ or $P = 4200$

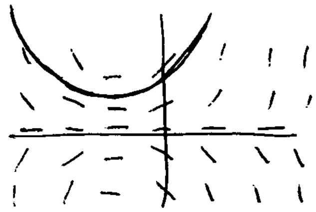
Sect 9.2

#3 $y' = 2 - y$ same on horizontal $\therefore$ III

#5 $y' = x + y - 1$ same on diagonals where $x + y = k$; at $x + y = 0$ ∴ slope is -1 or $y = -x$ $\Big\} \Rightarrow$ IV

Sect 9.2

#13 $y' = y + xy = (x+1)y$



Sect 9.3

#9 $\dfrac{du}{dt} = 2 + 2u + t + tu = (2+t)(1+u)$

$\dfrac{du}{1+u} = (2+t)\,dt \Rightarrow \ln(1+u) = 2t + \dfrac{t^2}{2} + c$

$1 + u = e^{2t + t^2/2 + c} = A\,e^{2t + \frac{t^2}{2}} \Rightarrow \boxed{u = Ae^{2t+\frac{t^2}{2}} - 1}$

A arb.

#12 $\dfrac{dy}{dx} = \dfrac{y\cos x}{1+y^2} \qquad y(0) = 1$

$\dfrac{(1+y^2)}{y}\,dy = \cos x\,dx \Rightarrow \int \frac{1}{y} + y\,dy = \int \cos x\,dx$

$\ln|y| + \dfrac{y^2}{2} = \sin x + c$

$y(0)=1 \qquad \ln(1) + \dfrac{1^2}{2} = \sin(0) + c \Rightarrow \dfrac{1}{2} = c$

$\boxed{\sin x = \ln y + \frac{y^2}{2} - \frac{1}{2}}$

$\boxed{x = \sin^{-1}\left[\ln y + \frac{y^2}{2} - \frac{1}{2}\right]}$

#15 $\dfrac{du}{dt} = \dfrac{2t + \sec^2 t}{2u} \qquad u(0) = -5$

$\int 2u\,du = \int 2t + \sec^2 t\,dt \Rightarrow u^2 = t^2 + \tan(t) + c \qquad u = \pm\sqrt{t^2 + \tan t + c}$

$-5 = -\sqrt{0 + 0 + c} \Rightarrow c = 25$ must be $-$ since $\sqrt{\ } \geq 0$

$\boxed{u = -\sqrt{t^2 + \tan(t) + 25}}$

Sect 9.4

#3 $\dfrac{dy}{dt} = ky\left(1 - \dfrac{y}{K}\right) \qquad K = 8\times10^7 \quad k = 0.71 \quad P(0) = 2\times10^7 \qquad y(t) = \dfrac{K}{1 + A\,e^{-kt}} \qquad A = \dfrac{K - P_0}{P_0}$

$A = \dfrac{8\times10^7 - 2\times10^7}{2\times10^7} = \dfrac{8-2}{2} = 3 \quad \therefore \quad \boxed{y(t) = \dfrac{8\times10^7}{1 + 3e^{-0.71t}}}$

$y(1) = \dfrac{8\times10^7}{1+3e^{-0.71}} = \dfrac{8\times10^7}{2.475} = \boxed{3.232\times10^7}$

$y(t) = 4\times10^7 = \dfrac{8\times10^7}{1+3e^{-0.71t}} \Rightarrow 1 + 3e^{-0.71t} = \dfrac{8\times10^7}{4\times10^7} = 2 \Rightarrow e^{-0.71t} = \dfrac{1}{3}$

$-0.71t = \ln\left(\frac{1}{3}\right) = -1.0986$

$t = \dfrac{1.0986}{0.71} = \boxed{1.547\text{ yr}}$

Sect 9.4 #5

1990 5.3 b
birth rate 35–40 m/yr. } ≈ $+\frac{20}{5300} = \frac{1}{265}$ per yr
death rate 15–20 m/yr
Carrying capacity (max) 100 b

(a) $P(t) = \frac{100}{1+Ae^{-kt}}$ $\frac{dP}{dt} = \frac{1}{265} P\left(1 - \frac{P}{100}\right)$

(b) $P(t) = \frac{100}{1+Ae^{-\frac{t}{265}}}$ $P(0) = \frac{100}{1+A} = 5.3 \Rightarrow A = 17.87$

$P(t) = \frac{100}{1+17.87e^{-\frac{t}{265}}}$ $P(10) = \frac{100}{1+17.87(0.963)} = \boxed{5.49}$ vs. actual $\boxed{6.1}$

(c) 2100 $t = 110$ $P(110) = \frac{100}{1+17.87(0.6603)} = \boxed{7.81 \text{ b}}$

2500. $t = 510$ $P(510) = \frac{100}{1+17.87(0.1459)} = \boxed{27.72 \text{ b}}$

(d) if $P(t) = \frac{50}{1+Ae^{-\frac{t}{265}}}$ $P(0) = \frac{50}{1+A} = 5.3 \Rightarrow A = 8.434$

now $P(t) = \frac{50}{1+8.434e^{-t/265}}$ $P(10) = \boxed{5.48}$, $P(110) = \boxed{7.61}$, $P(510) = \boxed{22.42}$

#7 (a) $\frac{dy}{dt} = ky(1-y)$ y = fraction who have heard rumor

(b) $y(t) = \frac{1}{1+Ae^{-kt}}$ $y(0) = \frac{1}{1+A} = y_0$ $A = \frac{1}{y_0} - 1$

$= \frac{1}{1+\left(\frac{1}{y_0}-1\right)e^{-kt}}$ or $\boxed{\frac{y_0}{y_0+(1-y_0)e^{-kt}}}$

(c) 8Am $\frac{80}{1000} = 0.08 = y_0$

NOON $\frac{500}{1000} = 0.5 = \frac{0.08}{0.08+0.92e^{-4k}}$

(k=4)

$0.08 = 0.04 + 0.46e^{-4k}$

$e^{-4k} = \frac{0.04}{0.46} = 0.086956\ldots$

$-4k = -2.4423\ldots$

$\boxed{k = 0.61059}$

$y(t) = \frac{0.08}{0.08+0.92e^{-0.61059t}} = 0.9$

$0.08 = 0.072 + 0.828e^{-0.61059t} \Rightarrow e^{-0.61059t} = 0.0096618\ldots \Rightarrow -0.61059t = -4.63957$

$\Rightarrow t = 7.6$ hr

8Am + 7.6 hr = 3.6 afternoon = $\boxed{3 \text{ hr } 36 \text{ min}}$

$\boxed{3{:}36 \text{ PM}}$

Sect 9.5

#5 $y' + 2y = 2e^x$ $\quad I = e^{\int 2\,dx} = e^{2x}$

$e^{2x}y' + 2ye^{2x} = \frac{d}{dt}(e^{2x}y) = 2e^{3x}$

$e^{2x}y = \int 2e^{3x}\,dx = \frac{2e^{3x}}{3} + c$ $\quad \boxed{y = \frac{2}{3}e^x + Ce^{-2x}}$

#9 $xy' + y = \sqrt{x}$ $\quad \left(I = e^{\int \frac{1}{x}dx} = e^{\ln x} = x\right)$ $\quad xy' + y = (xy)' = x^{1/2}$

$y' + \frac{1}{x}y = x^{-1/2}$

$xy = \int x^{1/2}dx = \frac{2}{3}x^{3/2} + c$

$\boxed{y = \frac{2}{3}\sqrt{x} + \frac{c}{x}}$

#15 $y' - y = x$ $\quad y(0) = 2$ $\quad \left(I = e^{\int -1\,dx} = e^{-x}\right)$

$e^{-x}y' - ye^{-x} = (e^{-x}y)' = xe^{-x}$ $\quad e^{-x}y = \int xe^{-x}dx = -xe^{-x} - e^{-x} + c$

$y = -x - 1 + Ce^{-x}$ $\quad y(0) = -1 + C = 2$ $\quad C = 3$ $\quad \boxed{y = -x - 1 + 3e^{-x}}$

#19 $xy' - y = x^2 \sin x$ $\quad y(\pi) = 0$ $\quad \left(I = e^{-\int \frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}\right)$

$y' - \frac{1}{x}y = x\sin x$ $\quad \frac{1}{x}y' - \frac{1}{x^2}y = \left(\frac{y}{x}\right)' = \sin x$ $\quad \frac{y}{x} = \int \sin x\,dx = -\cos x + C$

$y = -x\cos x + Cx$ $\quad y(\pi) = -\pi\cos\pi + C\pi = \pi + C\pi = 0 \Rightarrow C = -1$

$\boxed{y = -x\cos x - x}$

Sect 9.6

#1 (a) $\frac{dx}{dt} = -0.05x + 0.0001xy$

$\frac{dy}{dt} = 0.1y - 0.05xy$

If $y = 0$ $\frac{dx}{dt} = -0.05x$ which indicates when $y = 0$ pop. x declines $\Rightarrow$ X is a predator

$\therefore$ y is prey (note $x = 0 \Rightarrow \frac{dy}{dt} = +0.1y$)

The product terms xy represent interactions between prey & predator.

(b) $\frac{dx}{dt} = 0.2x - 0.0002x^2 - 0.006xy$

$\frac{dy}{dt} = -0.015y + 0.00008xy$

If $x = 0$ $\frac{dy}{dt} = -0.015y$ which indicates that y is a predator and X is a prey.