# Math 132 – Prof. Richard B. Goldstein – Stewart – 6ed

## Chapter 8

### Sect 8.1 #7

$y = 1 + 6x^{3/2} \quad 0 \le x \le 1$

$y' = 6\cdot\frac{3}{2}x^{1/2} = 9\sqrt{x}$

$\sqrt{1+(y')^2} = \sqrt{1+81x}$

$\Rightarrow L = \int_0^1 \sqrt{1+81x}\,dx$

let $u = 1+81x$, $u(0)=1$, $du = 81\,dx$, $u(1)=82$

$\frac{1}{81}\int_1^{82} u^{1/2}\,du = \frac{2}{243}u^{3/2}\Big|_1^{82} = \boxed{\frac{2}{243}\left(82\sqrt{82}-1\right)}$

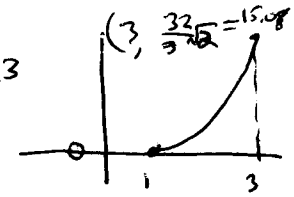
#11 $x = \frac{1}{3}y^{3/2} - y^{1/2} \qquad \frac{dx}{dy} = \frac{1}{2}y^{1/2} - \frac{1}{2}y^{-1/2}$

$1 \le y \le 9$

$1+\left(\frac{dx}{dy}\right)^2 = 1+\left(\frac{1}{2}y^{1/2}-\frac{1}{2}y^{-1/2}\right)^2 = 1+\frac{y}{4}-\frac{1}{2}+\frac{y^{-1}}{4} = \left(\frac{\sqrt{y}}{2}+\frac{1}{2\sqrt{y}}\right)^2$

$\int_1^9 \frac{\sqrt{y}}{2}+\frac{1}{2\sqrt{y}}\,dy = \left(\frac{y^{3/2}}{3}+y^{1/2}\right)\Big|_1^9 = (9+3)-\left(\frac{1}{3}+1\right) = \boxed{\frac{32}{3}}$

#21 $y = \frac{2}{3}(x^2-1)^{3/2} \quad 1 \le x \le 3$




$D = \sqrt{(3-1)^2+\left(\frac{32}{3}\sqrt{2}-0\right)^2} = \sqrt{231.555} = 15.2$

$y' = 2x\sqrt{x^2-1}$

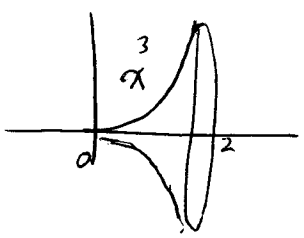
$1+(y')^2 = 4x^4-4x^2-1 = (2x^2-1)^2$

$L = \int_1^3 (2x^2-1)\,dx = \frac{2x^3}{3}-x\Big|_1^3 = 15-\left(-\frac{1}{3}\right) = 15.333\ldots = \boxed{\frac{46}{3}}$

### Sect 8.2

#5 $y = x^3 \quad 0 \le x \le 2$

$\frac{dy}{dx} = 3x^2 \qquad \left(\frac{dy}{dx}\right)^2 = 9x^4$




$S = \int_0^2 2\pi x^3\sqrt{1+9x^4}\,dx$

$u = 1+9x^4 \qquad du = 36x^3\,dx \qquad \frac{1}{36}du = x^3\,dx$

$\int_1^{145} \frac{2\pi}{36}u^{1/2}\,du$

$S = \frac{\pi}{18}\cdot\frac{u^{3/2}}{(3/2)}\Big|_1^{145} = \frac{\pi}{27}\left(145\sqrt{145}-1\right)$

#11 $x = \frac{1}{3}(y^2+2)^{3/2} \qquad 1 \le y \le 2$

$\frac{dx}{dy} = \frac{1}{2}(y^2+2)^{1/2}\cdot 2y = y\sqrt{y^2+2}$

$1+\left(\frac{dx}{dy}\right)^2 = 1+y^4+2y^2 = (y^2+1)^2$

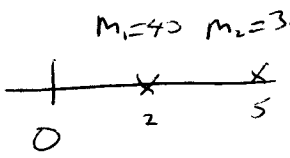
$S = \int_1^2 2\pi\, y\,(y^2+1)\,dy$

$= \pi\int_1^2 2y^3+2y\,dy = \pi\left[\frac{y^4}{2}+y^2\right]_1^2$

$= \pi\left(12-\frac{3}{2}\right) = \boxed{\frac{21\pi}{2}}$

## Sect 8.3

#21




$m = 40+30 = 70$

$M = \sum m_i x_i = 2(40)+5(30) = \boxed{230}$ $\quad \bar{x} = \frac{230}{70} = \boxed{\frac{23}{7}}$

#23 $m_1 = 6$ $P_1: (1,5)$

$m_2 = 5$ $P_2: (3,-2)$

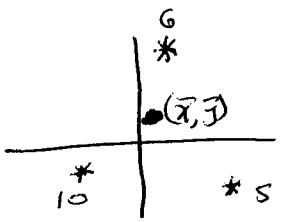
$m_3 = 10$ $P_3 (-2,-1)$

$m = 21$

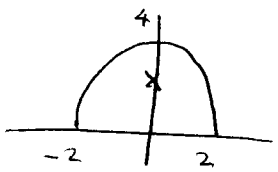
$M_y = \sum m_i x_i = 6(1)+5(3)+10(-2) = \boxed{1}$

$m_x = \sum m_i y_i = 6(5)+5(-2)+10(-1) = \boxed{10}$

$(\bar{x}, \bar{y}) = \left(\frac{1}{21}, \frac{10}{21}\right)$




#25 $y = 4-x^2$, $y = 0$




clearly $\bar{x} = 0$ by symmetry

$$\bar{y} = \frac{\int_{-2}^{2} \frac{1}{2}(4-x^2)^2\,dx}{\int_{-2}^{2}(4-x^2)\,dx} = \frac{\left(\frac{x^5}{10} - \frac{4x^3}{3} + 8x\right)\Big|_{-2}^{2}}{\left(4x - \frac{x^3}{3}\right)\Big|_{-2}^{2}} = \frac{256/15}{32/3} = \boxed{1.6}$$

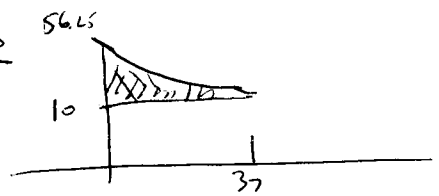
## Sect 8.4

#3 $C' = 74 + 1.1x - 0.002x^2 + 0.00004x^3$

$$C \text{ incr} = \int_{1200}^{1600} C'\,dx = \int_{1200}^{1600} 74 + 1.1x - 0.002x^2 + 0.00004x^3\,dx$$

$$= 74x + 0.55x^2 - \frac{0.002}{3}x^3 + 0.00001x^4\Big|_{1200}^{1600} = 64{,}331{,}733.33 - (20{,}464{,}800) = \boxed{43{,}866{,}933.33}$$

#5 $p = \frac{450}{x+8}$




vs. $8 \le \cdot \le 10$

$\frac{450}{x+8} = 10$ $\quad 45 = x+8 \Rightarrow x = 37$

$$\int_0^{37} \frac{450}{x+8} - 10\,dx = 450\ln(x+8) - 10x\Big|_0^{37} = (450\ln 45 - 370) - 450\ln 8 \approx \boxed{407.25}$$

#17 $c(t) = 20te^{-0.6t}$ $\quad 0 \le t \le 10$

$$\int_0^{10} 20te^{-0.6t}\,dt = -\frac{100}{9}e^{-0.6t}(3t+5)\Big|_0^{10} = 54.592$$

$$\frac{6\ L}{54.592\ sec} \times 60\ \frac{sec}{min} = \boxed{6.594\ \frac{L}{min}}$$

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## Sect 8.5

#3 $f(x) = \frac{3}{64} x\sqrt{16-x^2} \quad 0 \le x \le 4$ $\int \frac{3}{64} x(16-x^2)^{1/2} dx$

let $u = 16-x^2$
$du = -2x\,dx$
$-\frac{1}{2}du = x\,dx$

$= \frac{3}{64}\left(-\frac{1}{2}\right)\int u^{1/2} du = \left(-\frac{u^{3/2}}{64}\right)$

$u(0) = 16$
$u(4) = 0$
$u(12) = 12$

$\int_0^4 f(x)dx = \left(-\frac{u^{3/2}}{64}\right)\Big|_{16}^{0} = 0 - \left(-\frac{64}{64}\right) = 1$ ✓

$\int_0^2 f(x)dx = \left(-\frac{u^{3/2}}{64}\right)\Big|_{16}^{12} = 1 - \frac{12\sqrt{12}}{64} = \boxed{1 - \frac{3\sqrt{3}}{8} = 0.3505}$

#11 $\mu = 2.5$ min $\quad f(t) = \frac{e^{-\frac{t}{2.5}}}{2.5} \quad F(t) = 1 - e^{-\frac{t}{2.5}} = 1 - e^{-0.4t}$

(a) $P(T \ge 4) = e^{-\frac{4}{2.5}} = \boxed{e^{-1.6} \approx 0.20}$

(b) $P(T \le 2) = 1 - e^{-\frac{2}{2.5}} = 1 - e^{-0.8} \approx \boxed{0.55}$

(c) $e^{-0.4t} = 0.02 \quad t = \frac{\ln(0.02)}{-0.4} = 9.78 \sim \boxed{10 \text{ mins}}$

#13 $\mu = 9.4$ lb
$\sigma = 4.2$ lb.

$P(X \ge 10) = P\left(z \ge \frac{10-9.4}{4.2}\right) = \boxed{P(z \ge 0.143) \simeq 0.44}$ use tables