Math 132 – Prof. Richard B. Goldstein – Stewart – 6ed

Chapter 7

Sect 7.1 #7

$$\int x^2 \sin \pi x \, dx \qquad \begin{matrix} u = x^2 & dv = \sin \pi x \, dx \\ du = 2x\,dx & v = -\frac{\cos \pi x}{\pi} \end{matrix} \Rightarrow -\frac{x^2 \cos \pi x}{x} - \int -\frac{\cos \pi x}{\pi} 2x \, dx$$

$$= -\frac{x^2 \cos \pi x}{\pi} + \frac{2}{\pi}\int x \cos \pi x \, dx \qquad \begin{matrix} u = x & dv = \cos \pi x \, dx \\ du = dx & v = \frac{\sin \pi x}{\pi} \end{matrix} \Rightarrow -\frac{x^2 \cos \pi x}{\pi} + \frac{2}{\pi}\left\{ \frac{x \sin \pi x}{\pi} - \int \frac{\sin \pi x}{\pi} dx \right\}$$

$$= -\frac{x^2 \cos \pi x}{\pi} + \frac{2x \sin \pi x}{\pi^2} - \frac{2}{\pi^2}\left(\frac{-\cos \pi x}{\pi}\right) = \boxed{-\frac{x^2 \cos \pi x}{\pi} + \frac{2x \sin \pi x}{\pi^2} + \frac{2 \cos \pi x}{\pi^3}}$$

#9

$$\int \ln(2x+1)\, dx \qquad \begin{matrix} u = \ln(2x+1) & dv = dx \\ du = \frac{2}{2x+1} dx & v = x \end{matrix} \Rightarrow x \ln(2x+1) - \int \frac{2x}{2x+1} dx$$

$$= x\ln(2x+1) - \int \left(1 - \frac{1}{2x+1}\right) dx = \boxed{x \ln(2x+1) - x + \frac{\ln(2x+1)}{2}}$$

#17

$$\int e^{2\theta} \sin 3\theta \, d\theta \qquad \begin{matrix} u = e^{2\theta} & dv = \sin 3\theta \, d\theta \\ du = 2e^{2\theta} d\theta & v = -\frac{\cos 3\theta}{3} \end{matrix} \Rightarrow -\frac{e^{2\theta}\cos 3\theta}{3} - \int -\frac{\cos 3\theta}{3}(2) e^{2\theta} d\theta$$

$$= -\frac{e^{2\theta}\cos 3\theta}{3} + \frac{2}{3}\int e^{2\theta} \cos 3\theta \, d\theta \qquad \begin{matrix} u = e^{2\theta} & dv = \cos 3\theta \, d\theta \\ du = 2e^{2\theta} d\theta & v = \frac{\sin 3\theta}{3} \end{matrix}$$

$$= -\frac{e^{2\theta}\cos 3\theta}{3} + \frac{2}{3}\left(e^{2\theta}\frac{\sin 3\theta}{3} - \int \frac{\sin 3\theta}{3}(2) e^{2\theta} d\theta\right)$$

original integral

$$= -\frac{e^{2\theta}\cos 3\theta}{3} + \frac{2e^{2\theta}\sin 3\theta}{9} - \frac{4}{9}\int e^{2\theta} \sin 3\theta \, d\theta$$

$$\therefore \frac{13}{9}(I) = -\frac{e^{2\theta}\cos 3\theta}{3} + \frac{2e^{2\theta}\sin 3\theta}{9} \Rightarrow I = \boxed{-\frac{9e^{2\theta}\cos 3\theta}{39} + \frac{2e^{2\theta}\sin 3\theta}{13}}$$

#35

$$\int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^3 \cos(\theta^2)\, d\theta \qquad \text{let } x = \theta^2, \quad dx = 2\theta \, d\theta, \quad x\left(\sqrt{\tfrac{\pi}{2}}\right) = \tfrac{\pi}{2}, \quad x(\sqrt{\pi}) = \pi \Rightarrow \frac{1}{2}\int_{\pi/2}^{\pi} x \cos(x)\, dx \qquad \begin{matrix} u = x \\ du = dx \\ dv = \cos x \, dx \\ v = \sin x \end{matrix}$$

$$\left(\frac{1}{2} x \sin x\right)\Big|_{\pi/2}^{\pi} - \frac{1}{2}\int_{\pi/2}^{\pi} \sin x \, dx = \left(0 - \frac{\pi}{4}\right) + \frac{1}{2} \cos x \Big|_{\pi/2}^{\pi} = -\frac{\pi}{4} + \frac{1}{2}(-1 - 0)$$

$$= \boxed{-\frac{\pi}{4} - \frac{1}{2}}$$

Sect 7.2

Math 132 ②
Prof. Richard B Goldstein
Stewart 6ed

#3 $\displaystyle\int_{\pi/2}^{3\pi/4} \sin^5 x \cos^3 x \, dx$

use $\cos^2 x = 1 - \sin^2 x$

$\frac{d}{dx} \sin x = \cos x$

$$= \int_{\pi/2}^{3\pi/4} \sin^5 x \,(1 - \sin^2 x)\, d(\sin x)$$

let $u = \sin x$

$u(\frac{\pi}{2}) = 1$

$u(\frac{3\pi}{4}) = \frac{\sqrt{2}}{2}$

$$\int_1^{\sqrt{2}/2} u^5 - u^7 \, du = \left(\frac{u^6}{6} - \frac{u^8}{8}\right)\Bigg|_1^{\sqrt{2}/2}$$

$$= \left(\frac{1}{48} - \frac{1}{128}\right) - \left(\frac{1}{6} - \frac{1}{8}\right) = \boxed{-\frac{11}{384}}$$

#11 $$\int (1+\cos\theta)^2 d\theta = \int 1 + 2\cos\theta + \cos^2\theta \, d\theta = \int 1 + 2\cos\theta + \frac{1}{2}(1 + \cos 2\theta)\, d\theta$$

$$= \theta + 2\sin\theta + \frac{\theta}{2} + \frac{\sin 2\theta}{4} = \boxed{\frac{3\theta}{2} + 2\sin\theta + \frac{\sin 2\theta}{4}}$$

#29 $\displaystyle\int \tan^3 x \sec x \, dx$

$\tan^2 x = \sec^2 x - 1$

$\frac{d}{dx} \sec x = \sec x \tan x$

$$\int (\sec^2 x - 1) \sec x \tan x \, dx = \boxed{\frac{\sec^3 x}{3} - \sec x}$$

Sect 7.3

#5 $\displaystyle\int_{\sqrt{2}}^{2} \frac{1}{t^3\sqrt{t^2-1}}\, dt$

$t = \sec\theta$

$\sqrt{t^2-1} = \tan\theta$

$dt = \sec\theta \tan\theta \, d\theta$

$\sec\theta = 2 \quad \cos\theta = \frac{1}{2} \quad \theta = \frac{\pi}{3}$

$\sec\theta = \sqrt{2} \quad \cos\theta = \frac{1}{\sqrt{2}} \quad \theta = \frac{\pi}{4}$

$$= \int_{\pi/4}^{\pi/3} \frac{1}{\sec^3\theta \tan\theta} \sec\theta \tan\theta \, d\theta = \int_{\pi/4}^{\pi/3} \frac{1}{\sec^2\theta} d\theta = \int_{\pi/4}^{\pi/3} \cos^2\theta \, d\theta = \int_{\pi/4}^{\pi/3} \frac{1 + \cos 2\theta}{2} d\theta$$

$$= \frac{\theta}{2} + \frac{\sin 2\theta}{4}\Bigg|_{\pi/4}^{\pi/3} = \left(\frac{\pi}{6} + \frac{\sqrt{3}}{8}\right) - \left(\frac{\pi}{8} + \frac{1}{4}\right) = \boxed{\frac{\pi}{24} + \frac{\sqrt{3}}{8} - \frac{1}{4}}$$

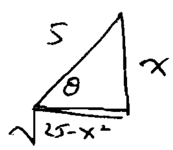
#7 $\displaystyle\int \frac{1}{x^2\sqrt{25-x^2}}\, dx$

$x = 5\sin\theta$

$dx = 5\cos\theta \, d\theta$

$\sqrt{25-x^2} = 5\cos\theta$

$$\int \frac{1}{25\sin^2\theta \,(5\cos\theta)} (5\cos\theta \, d\theta)$$



$$= \frac{1}{25}\int \csc^2\theta \, d\theta = -\frac{1}{25}\cot\theta = \boxed{-\frac{1}{25}\left(\frac{\sqrt{25-x^2}}{x}\right)}$$

Prof. R. B Goldstein
Stewart 6th Ed

Sect 7.4

#3a $\dfrac{x^4+1}{x^5+4x^3} = \dfrac{x^4+1}{x^3(x^2+4)} = \boxed{\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x^3}+\dfrac{Dx+E}{x^2+4}}$

#3b $\dfrac{1}{(x^2-9)^2} = \dfrac{1}{(x-3)^2(x+3)^2} = \boxed{\dfrac{A}{x-3}+\dfrac{B}{(x-3)^2}+\dfrac{C}{x+3}+\dfrac{D}{(x+3)^2}}$

#9 $\displaystyle\int \frac{x-9}{(x+5)(x-2)}\,dx$ $\qquad \dfrac{x-9}{(x+5)(x-2)} = \dfrac{A}{x+5}+\dfrac{B}{x-2}$

$x-9 = A(x-2)+B(x+5)$

at $x=2$ $\quad -7 = 7B \quad B=-1$

at $x=-5$ $\quad -14=-7A \quad A=2$

$\displaystyle\int \frac{2}{x+5} - \frac{1}{x-2}\,dx = \boxed{2\ln|x+5| - \ln|x-2|}$

#17 $\displaystyle\int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)}\,dy$ $\qquad \dfrac{4y^2-7y-12}{y(y+2)(y-3)} = \dfrac{A}{y}+\dfrac{B}{y+2}+\dfrac{C}{y-3}$

$4y^2-7y-12 = A(y+2)(y-3)+By(y-3)+Cy(y+2)$

at $y=0$ $\quad -12=-6A \quad A=2$

$y=-2$ $\quad 18=10B \quad B=1.8$

$y=3$ $\quad 3=15C \quad C=0.2$

$\displaystyle\int_1^2 \frac{2}{y}+\frac{1.8}{y+2}+\frac{0.2}{y-3}\,dy = 2\ln|y| + 1.8\ln|y+2| + 0.2\ln|y-3|\Big]_1^2$

$= (2\ln 2 + \underbrace{1.8\ln 4}_{3.6\ln 2} + 0) - (0 + 1.8\ln 3 + 0.2\ln 2) = \boxed{5.4\ln 2 - 1.8\ln 3}$

#25 $\displaystyle\int \frac{10}{(x-1)(x^2+9)}\,dx$ $\qquad \dfrac{10}{(x-1)(x^2+9)} = \dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+9}$

$10 = A(x^2+9) + Bx(x-1) + C(x-1)$

at $x=1$ $\quad 10=10A \Rightarrow \underline{A=1}$

at $x=0$ $\quad 10 = 9A - C = 9 - C \Rightarrow \underline{C=-1}$

(arb.) at $x=-1$ $\quad 10 = 10A + 2B - 2C \qquad 10 = 10 + 2B + 2 \qquad \underline{B=-1}$

$\displaystyle\int \frac{1}{x-1} - \frac{1}{x^2+9} - \frac{x}{x^2+9}\,dx = \boxed{\ln|x-1| - 3\arctan\left(\frac{x}{3}\right) - \frac{1}{2}\ln(x^2+9)}$

Sect 7.5

Math 132 (4)

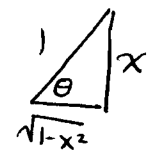
Prof. R.B. Goldstein

Stewart 6th Ed

\#7 $\displaystyle\int_{-1}^{+1} \frac{e^{\arctan y}}{1+y^2}\,dy$ let $y=\tan x$, $x=\arctan y$, $dy=\sec^2 x\,dx$ $\qquad \arctan 1 = \frac{\pi}{4}$, $\arctan(-1) = -\frac{\pi}{4}$

$$\int_{-\pi/4}^{\pi/4} \frac{e^x}{\sec^2 x}\sec^2 x\,dx = \int_{-\pi/4}^{\pi/4} e^x\,dx = e^x\Big|_{-\pi/4}^{\pi/4} = \boxed{e^{\pi/4} - e^{-\pi/4}}$$

\#15 $\displaystyle\int \frac{dx}{(1-x^2)^{3/2}}$ $\quad x=\sin\theta$, $dx=\cos\theta\,d\theta$ $\quad \displaystyle\int \frac{\cos\theta\,d\theta}{\cos^3\theta} = \int \sec^2\theta\,d\theta = \tan\theta = \boxed{\frac{x}{\sqrt{1-x^2}}}$



\#35 $\displaystyle\int_{-1}^{+1} x^8 \sin x\,dx$ Since $x^8\sin x$ is odd on $[-1,1]$ the integral is $\boxed{0}$

Sect 7.6

\#1 $\displaystyle\int \frac{\sqrt{7-2x^2}}{x^2}\,dx$ $\qquad$ \#33 $\displaystyle\int \frac{\sqrt{a^2-u^2}}{u^2}\,du = -\frac{1}{u}\sqrt{a^2-u^2} - \sin^{-1}\left(\frac{u}{a}\right)$

Use $a=7$ $\quad u=\sqrt{2}\,x$ $\quad u^2=2x^2$ $\quad x^2=\frac{1}{2}u^2$ $\quad dx=\frac{du}{\sqrt{2}}$ $\qquad \displaystyle\int \frac{\sqrt{7-u^2}}{\frac{1}{2}u^2}\frac{du}{\sqrt{2}} = \sqrt{2}\int \frac{\sqrt{7-u^2}}{u^2}\,du$

$$= \sqrt{2}\left[-\frac{1}{u}\sqrt{7-u^2} - \sin^{-1}\left(\frac{u}{\sqrt{7}}\right)\right] = \sqrt{2}\left[-\frac{\sqrt{7-2x^2}}{\sqrt{2}\,x} - \sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{7}}\right)\right]$$

$$= \boxed{-\frac{1}{x}\sqrt{7-2x^2} - \sqrt{2}\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{7}}\right)}$$

\#3 $\displaystyle\int \sec^3(\pi x)\,dx$ $\qquad$ \#71 $\displaystyle\int \sec^3 u\,du = \frac{1}{2}\sec u\tan u + \frac{1}{2}\ln|\sec u + \tan u|$

$u=\pi x$, $du=\pi\,dx$

$$\boxed{\frac{1}{2\pi}\sec(\pi x)\tan(\pi x) + \frac{1}{2\pi}\ln|\sec(\pi x)+\tan(\pi x)|}$$

Sect 7.7

\#9 $\displaystyle\int_1^2 \frac{\ln x}{1+x}\,dx$

x	1	1.1	1.2	1.3	1.4	1.5
$f(x)$	0	0.045386	0.082873	0.114071	0.140197	0.162186

1.6	1.7	1.8	1.9	2
0.180771	0.196529	0.209924	0.221329	0.231049

$$T_{10} = \frac{0.1}{2}\left[f(1)+2f(1.1)+2f(1.2)+\ldots+2f(1.8)+2f(1.9)+f(2)\right] = \frac{0.1}{2}(2.937581) = \boxed{0.146879}$$

Math 132 (5)
Prof. R.B. Goldstein
Stewart 6^{th} Ed

$$M_{10} = 0.1\left[f(1.05) + f(1.15) + \dots + f(1.95)\right]$$
$$= 0.1\left[0.023800 + 0.065006 + \dots + 0.226383\right]$$
$$= 0.1\left[1.450114\right] = \boxed{0.145011}$$

$$S_{10} = \frac{0.1}{3}\left[f(1) + 4f(1.1) + 2f(1.2) + 4f(1.3) + \dots + 4f(1.9) + f(2)\right]$$
$$= \frac{0.1}{3}\left[4.416583\right] = \boxed{0.147219}$$

Sect 7.8

#5 $\displaystyle\int_1^{\infty} \frac{1}{(3x+1)^2}\,dx$

$u = 3x+1$
$du = 3\,dx$
$u(1) = 4$
$u(\infty) = \infty$
$du = 3\,dx$

$$\frac{1}{3}\int_4^{\infty} u^{-2}\,du = -\frac{1}{3}u^{-1}\Big|_4^{\infty} = 0 - \left(-\frac{1}{12}\right) = \boxed{\frac{1}{12}}$$

as a limit

#13 $$\int_{-\infty}^{\infty} x e^{-x^2}\,dx = \int_{-\infty}^{0} x e^{-x^2}\,dx + \int_0^{\infty} x e^{-x^2}\,dx$$

note xe^{-x^2} is an odd function
however to be 0 $\displaystyle\int_0^{\infty} x e^{-x^2}\,dx$ must exist

as a limit

Since $$\int_0^{\infty} x e^{-x^2}\,dx = \int_0^{\infty} \frac{e^{-u}}{2}\,du = -\frac{e^{-u}}{2}\Big|_0^{\infty} = 0 - \left(-\frac{1}{2}\right) = \frac{1}{2}$$

The left integral is $\left(-\frac{1}{2}\right)$ and right is $\left(\frac{1}{2}\right)$ ∴ $\boxed{0}$

#29 $\displaystyle\int_{-2}^{14} \frac{dx}{\sqrt[4]{x+2}}$

$du = dx$
$u = x+2$
$u(-2) = 0$
$u(14) = 16$

$$\int_0^{16} u^{-1/4}\,du = \frac{4}{3}u^{3/4}\Big|_0^{16} = \frac{4}{3}(8 - 0) = \boxed{\frac{32}{3}}$$

as a limit

#57 $$\int_0^1 \frac{1}{x^p}\,dx = \int_0^1 x^{-p}\,dx = \frac{x^{-p+1}}{-p+1}\Big|_0^1$$ $p<1$ OK

$$= \left(\frac{1}{-p+1}\right) - \left(\frac{0}{1}\right) = \boxed{\frac{1}{1-p}}$$

note $p=1$ $\displaystyle\int_0^1 \frac{1}{x}\,dx = \ln x\Big|_0^1$ does not exist

nor if $p>1$ ex. $\displaystyle\int_0^1 \frac{1}{x^3}\,dx = -\frac{x^{-2}}{2}\Big|_0^1$ D.N.E.