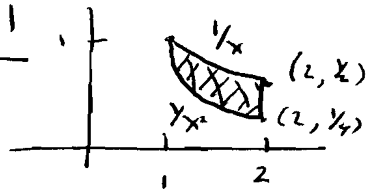
Math 132 - Prof. Richard B. Goldstein - Stewart _6ed

Sect 6.1

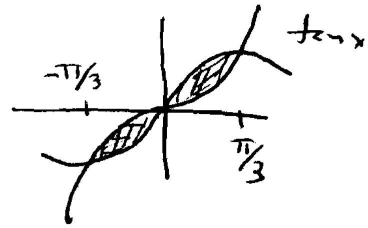
#9



$$\int_1^2 \frac{1}{x} - \frac{1}{x^2}\,dx = \left(\ln x + x^{-1}\right)\Big|_1^2 = (\ln 2 + 0.5) - (0+1)$$
$$= \boxed{\ln 2 - 0.5 \simeq 0.193}$$

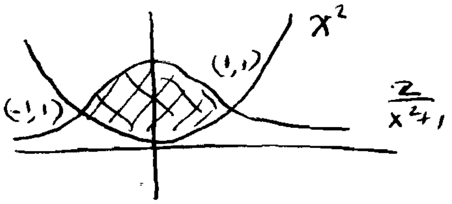
#15 $y = \tan x$, $y = 2\sin x$, $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$

note: $\tan\left(\frac{\pi}{3}\right) = 2\sin\left(\frac{\pi}{3}\right) = \sqrt{3}$
$\tan\left(-\frac{\pi}{3}\right) = 2\sin\left(-\frac{\pi}{3}\right) = -\sqrt{3}$



$$A = 2\int_0^{\pi/3} (2\sin x - \tan x)\,dx = 2\Big[-2\cos x + \ln(\cos x)\Big]_0^{\pi/3}$$
$$= 2\Big[\{-2(0.5) + \ln(0.5)\} - \{-2 + 0\}\Big] = 2\big[-1 + \ln(0.5) + 2\big] = \boxed{2 + 2\ln(0.5)}$$
$$\simeq \boxed{0.6137}$$

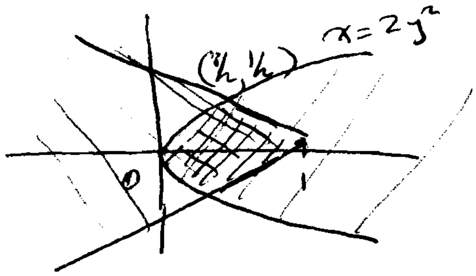
#25 $y = x^2$, $y = \frac{2}{x^2+1}$



$$A = 2\int_0^1 \frac{2}{x^2+1} - x^2\,dx$$
$$A = 2\left[2\arctan x - \frac{x^3}{3}\right]_0^1 = 2\left[\left(2\arctan 1 - \frac{1}{3}\right) - \left(2\arctan 0 - \frac{0}{3}\right)\right]$$
$$= 2\left[2\left(\frac{\pi}{4}\right) - \frac{1}{3}\right] = \boxed{\pi - \frac{2}{3}}$$

#40 $x - 2y^2 \ge 0$
$1 - x - |y| \ge 0$

$x \le 1 - |y|$

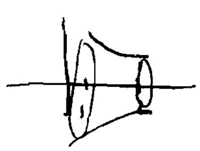


$A = 2$ (top piece)
$$= 2\int_0^{1/2} (1-y) - 2y^2\,dy$$
$1 - y - 2y^2$
$$= 2\left[y - \frac{y^2}{2} - \frac{2y^3}{3}\right]_0^{1/2} = 2\left[\frac{7}{24} - 0\right]$$
$$= \boxed{\frac{7}{12}}$$

A

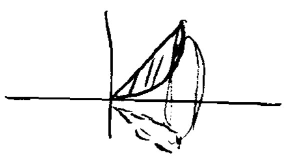
Sect 6.2

#3 $y = \frac{1}{x}$



$$V = \int_1^2 \pi\left(\frac{1}{x}\right)^2 dx = -\frac{\pi}{x}\Big|_1^2 = \boxed{\frac{\pi}{2}}$$

#7 $y = x^3$, $y = x$, $x \ge 0$



$$V = \pi\int_0^1 x^2 - x^6\,dx = \pi\left(\frac{x^3}{3} - \frac{x^7}{7}\right)_0^1 = \boxed{\frac{4\pi}{21}}$$

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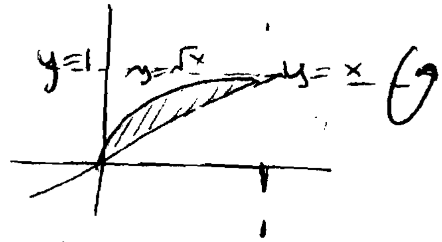

$$A(x) = \pi(1-x)^2 - \pi(1-\sqrt{x})^2$$
$$= \pi - 2\pi x + \pi x^2 - \pi + 2\pi\sqrt{x} - \pi x$$
$$= \pi(x^2 - 3x + 2\sqrt{x})$$

$$V = \pi\int_0^1 x^2 - 3x + 2\sqrt{x}\,dx = \pi\left[\frac{x^3}{3} - \frac{3x^2}{2} + \frac{2x^{3/2}}{3/2}\right]_0^1 = \pi\left(\frac{1}{6}\right) = \boxed{\frac{\pi}{6}}$$

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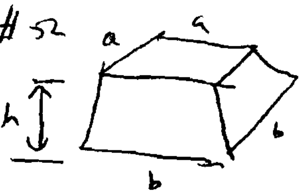

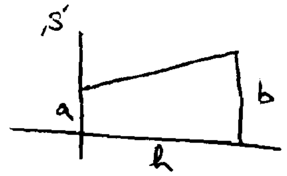

$$s = a + \frac{(b-a)x}{h} \qquad A = s^2 = \left[a + \left(\frac{b-a}{h}\right)x\right]^2$$

$$V = \int_0^h \left[a + \left(\frac{b-a}{h}\right)x\right]^2 dx = \int_0^h a^2 + \frac{2a(b-a)}{h}x + \frac{(b-a)^2}{h^2}x\,dx = \boxed{\frac{h(a^2+ab+b^2)}{3}}$$

if $a=b$, $V = \frac{h(3a^2)}{3} = ha^2$ ✓ if $a=0$ $V = \frac{hb^2}{3}$ ✓

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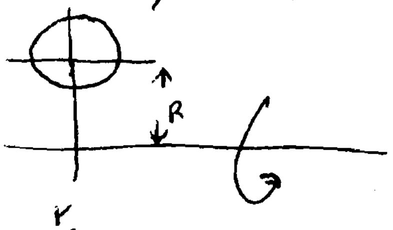

$$x^2 + (y-R)^2 = r^2 \Rightarrow \begin{array}{ll} y = R + \sqrt{r^2-x^2} & \text{upper (outer)} \\ y = R - \sqrt{r^2-x^2} & \text{lower (inner)} \end{array}$$

$$V = \pi\int_{-r}^{r} (R+\sqrt{r^2-x^2})^2 - (R-\sqrt{r^2-x^2})^2\,dx = 2\pi\int_0^r 4R\sqrt{r^2-x^2}\,dx = 8\pi R\int_0^r \sqrt{r^2-x^2}\,dx$$

$$= 8\pi R\left(\frac{\pi r^2}{4}\right) = \boxed{2\pi^2 r^2 R}$$

Sect 6.3

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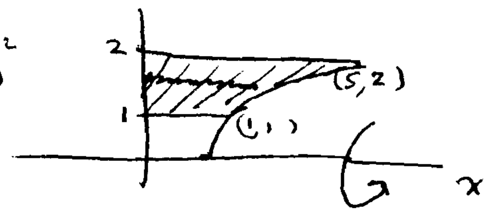
$x = 1 + y^2$
$x = 0$
$y = 1$
$y = 2$



$$2\pi\int_1^2 y(y^2+1)\,dy = 2\pi\int_1^2 (y^3+y)\,dy = 2\pi\left[\frac{y^4}{4} + \frac{y^2}{2}\right]_1^2$$
$$= 2\pi\left[6 - \frac{3}{4}\right] = \boxed{\frac{21\pi}{2}}$$

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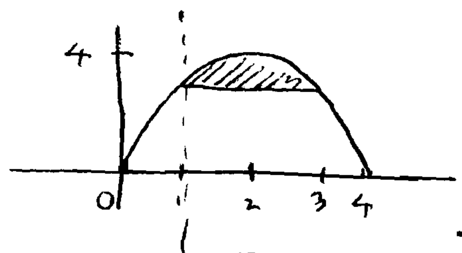
$y = 4x - x^2$
$y = 3$
about $x = 1$



$$V = 2\pi\int_1^3 (x-1)(4x-x^2-3)\,dx = 2\pi\int_1^3 (-x^3+5x^2-7x+3)\,dx$$
$$= 2\pi\left[-\frac{x^4}{4} + \frac{5x^3}{3} - \frac{7x^2}{2} + 3x\right]_1^3 = 2\pi\left[\frac{9}{4} - \frac{11}{12}\right] = \boxed{\frac{8\pi}{3}}$$

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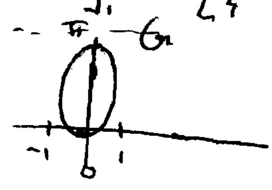
$$\left.\begin{array}{l} x = \sqrt{\sin y} \\ 0 \le y \le \pi \\ x = 0 \end{array}\right\} \text{about } y = 4$$

$$\boxed{V = \int_0^\pi 2\pi(4-y)\sqrt{\sin y}\,dy}$$



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Sect 6.4

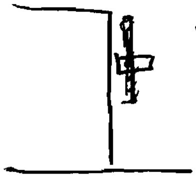
#3 $W = \int_0^9 \frac{10}{(1+x)^2}dx$ $\quad u = 1+x,\; du = dx$ $\quad \int_1^{10} 10u^{-2}\,du = -\frac{10}{u}\Big|_1^{10} = (-1+10) = \boxed{9 \text{ ft-lb}}$

#11 $f(x) = kx$ $\quad W_1 = \int_0^{10} kx\,dx = \frac{kx^2}{2}\Big|_0^{10} = 50k$

$W_2 = \int_{10}^{20} kx\,dx = \frac{kx^2}{2}\Big|_{10}^{20} = 200k - 50 = 150k$

3 times as much $\boxed{W_2 = 3W_1}$

#13



(a) $50(0.5) = 25$ lb

$\int_0^{50} (0.5)(x)\,dx = 0.25x^2\Big|_0^{50} = \boxed{625 \text{ ft-lb}}$

(b) 1st 25 ft $\quad \int_0^{25} 0.5x\,dx = 0.25x^2\Big|_0^{25} = 156.25$

2nd 25 ft $\quad \int_0^{25} 12.5\,dx = 312.5$

$= \boxed{468.75 \text{ ft-lb}}$

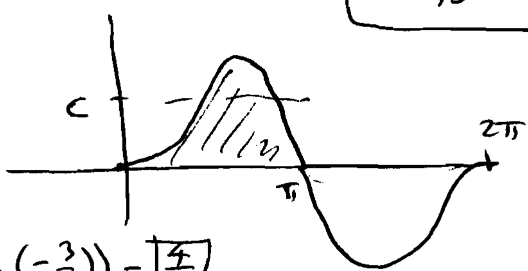
Sect 6.5

#3 $g_{avg} = \frac{\int_1^8 x^{1/3}\,dx}{8-1} = \frac{x^{4/3}}{28/3}\Big|_1^8 = \frac{16-1}{28/3} = \boxed{\frac{45}{28}}$

#5 $f_{avg} = \frac{\int_0^5 t e^{-t^2}\,dt}{5-0}$ $\quad$ let $u = t^2$, $du = 2t\,dt$, $\frac{du}{2} = t\,dt$

$\frac{1}{10}\int_0^{25} e^{-u}\,du = -\frac{e^{-u}}{10}\Big|_0^{25} = \boxed{\frac{(1-e^{-25})}{10}}$

#11 $f(x) = 2\sin x - \sin 2x \quad [0, \pi]$



(a) $f_{avg} = \frac{1}{\pi}\int_0^{\pi} (2\sin x - \sin 2x)\,dx$

$= \frac{1}{\pi}\left[-2\cos x + \frac{\cos 2x}{2}\right]_0^{\pi} = \frac{1}{\pi}\left(\frac{5}{2} - \left(-\frac{3}{2}\right)\right) = \boxed{\frac{4}{\pi}}$

(b) $2\sin x - \sin 2x = \frac{4}{\pi}$ by Derive $\quad \boxed{x = 1.238\ldots \text{ or } x = 2.808\ldots}$