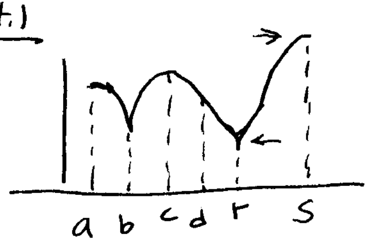
Math 131 - Prof. Richard B. Goldstein - Chap 4 HW - Stewart 6e

Sect 4.1

#3

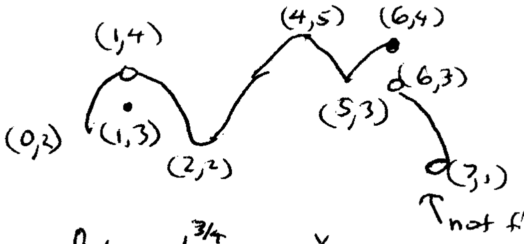


<u>abs max</u> s <u>local max</u> c

<u>abs min</u> r <u>local min</u> b, r

<u>neither</u> a max or min at a, d

#5



<u>abs max</u> $(4,5)$ <u>local max</u> $(4,5)$, $(6,4)$

<u>abs min</u> <u>none</u>   <u>Local min</u> $(2,2)$, $(1,3)$, $(5,3)$

#37 $h(t) = t^{3/4} - 2t^{1/4}$

$h'(t) = \frac{3}{4}t^{-1/4} - 2\left(\frac{1}{4}\right)t^{-3/4} = t^{-3/4}\left(\frac{3}{4}t^{1/2} - \frac{1}{2}\right) = 0$

$\frac{3}{4}t^{1/2} = \frac{1}{2}$

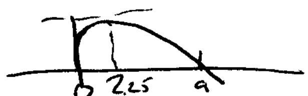
$t^{1/2} = 2/3$

$t = 4/9$

$h'(0)$ D.N.E.   <u>critical number $t = 4/9$</u>   (also $t=0$)

Sect 4.2

#3 $f(x) = \sqrt{x} - \frac{1}{3}x$ on $[0, 9]$

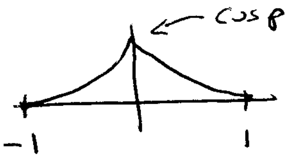


Continuous on $[0,9]$ $f'(x) = \frac{1}{2}x^{-1/2} - \frac{1}{3}$ derivate exists on $(0, 9)$

$f(0) = 0 - 0 = 0$ $f(9) = 3 - 3 = 0$

$f'(x) = 0 \Rightarrow \frac{1}{2\sqrt{x}} - \frac{1}{3} = 0 \Rightarrow \sqrt{x} = \frac{3}{2}$ $x = \frac{9}{4} \in (0,9)$ only 1 pt.

#5 $f(x) = 1 - x^{2/3}$ $f(-1) = f(1) = 0$



$f'(x) = -\frac{2}{3}x^{-1/3}$ $f'(x)$ D.N.E. at $x=0 \in [-1,1]$

$\therefore$ conditions for Rolle's Theorem not satisfied

#11 $f(x) = 3x^2 + 2x + 5$ on $[-1, 1]$ MVT.

$f(-1) = 3 - 2 + 5 = 6$

$f(1) = 3 + 2 + 5 = 10$

$\frac{f(1) - f(-1)}{1-(-1)} = \left(\frac{10-6}{1-(-1)}\right) = \frac{4}{2} = 2$

$f'(0) = 2$ ✓

$f'(x) = 6x + 2$ $6x + 2 = 2 \Rightarrow \underline{x = 0}$

#19 $f(x) = x^3 - 15x + c = 0$ has at most one root in $[-2, 2]$

if $-22 \leq c \leq 22$ there is only one root

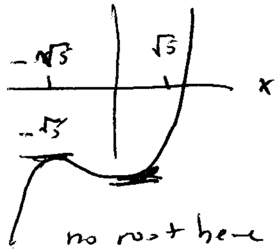
otherwise no roots

$f(-2) = -8 + 30 + c = 22 + c$

$f(2) = 8 - 30 + c = -22 + c$

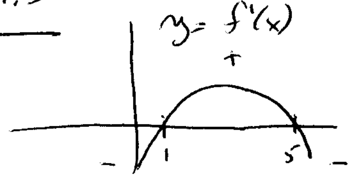
$f'(x) = 3x^2 - 15 = 0 \Rightarrow x^2 = 5 \Rightarrow x = \pm\sqrt{5}$ not in $[-2, 2]$

$\therefore$ no <u>local</u> <u>max</u> or <u>min</u> in $[-2, 2]$



Sect 4.3

#5

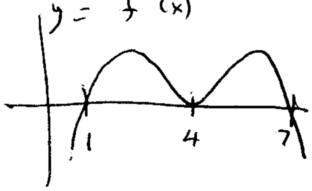


(a) f incr. for $(1,5)$
f decr for $(0,1)$ and $(5,6)$

(b) $-\ \underset{0}{\_}\ +$ local min at $x=1$

$+\ \overset{0}{\frown}\ -$ local max at $x=5$

#7



f'' changes sign at $x=1$ and $x=7$ and these are inflection points

#21 $f(x) = x + \sqrt{1-x}$

$f'(x) = 1 + \frac{1}{2}(1-x)^{-1/2}(-1) = 1 - \frac{1}{2\sqrt{1-x}} = 0 \Rightarrow \sqrt{1-x} = \frac{1}{2} \Rightarrow 1-x = \frac{1}{4} \Rightarrow x = \frac{3}{4}$ is a critical pt

$f'(\frac{3}{4}^-)$ ex. $f'(0.7) = 1 - \frac{1}{2\sqrt{.3}} = 0.087 > 0$ $f'(\frac{3}{4}^+)$ ex $f'(0.8) = 1 - \frac{1}{2\sqrt{.2}} = -0.118 < 0$ $\wedge$

$f''(x) = \frac{d}{dx}\left[1 - \frac{1}{2}(1-x)^{-1/2}\right] = \frac{1}{4}(1-x)^{-3/2}\cdot(-1) = -\frac{1}{4(1-x)^{3/2}}\Big|_{3/4} = -\frac{1}{4(\frac{1}{4})^{3/2}} = -2 < 0$

by both the first + second derivative test $x = 3/4$ is $\boxed{\text{a local max}}$

$f(3/4) = \frac{3}{4} + \frac{1}{2} = 5/4$

#33 $f(x) = 2x^3 - 3x^2 - 12x$

$f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x-2)(x+1) = 0$ at $x = -1, 2$ f' $+\ \overset{0}{|}\ -\ \overset{0}{|}\ +$ at -1, 2

$f''(x) = 12x - 6 = 0$ at $x = \frac{1}{2}$ f'' $-\ \overset{0}{|}\ +$ at $\frac{1}{2}$

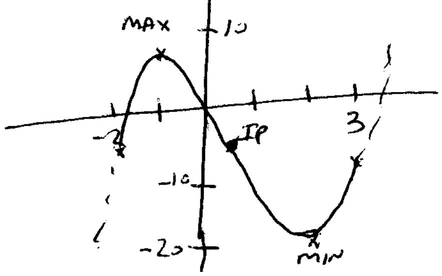
(a) f(x) increases for $(-\infty,-1)$ and $(2,\infty)$; decreases for $(-1,2)$

(b) local max at $x=-1$; local min at $x=2$

(c) concave up for $(\frac{1}{2},\infty)$; down for $(-\infty,\frac{1}{2})$; infl. pt at $x=\frac{1}{2}$

(d)

x	f(x)
−2	−4
−1	+7
0	0
½	−6.5
2	−20
3	−9



Sect 4.4

#7 $\lim\limits_{x\to1} \frac{x^9-1}{x^5-1}$ 0/0 $\overset{L'H}{=} \lim\limits_{x\to1} \frac{9x^8}{5x^4} = \boxed{\frac{9}{5}}$

#13 $\lim\limits_{x\to0} \frac{\tan px}{\tan qx}$ 0/0 $\overset{L'H}{=} \lim\limits_{x\to0} \frac{(\sec^2 px)\,p}{(\sec^2 qx)\,q} = \frac{1\cdot p}{1\cdot q} = \boxed{\frac{p}{q}}$

Math 131 (3)
Prof. R.B. Goldstein
Chap 4 HW
Stewart 6th Ed

Sect 4.4

#17 $\lim_{x \to 0^+} \frac{\ln x}{x}$ is $\frac{\infty}{0}$ for example $f(0.01) = \frac{-4.605}{.01} \to -\infty$

#25 $\lim_{t \to 0} \frac{5^t - 3^t}{t} \overset{L'H}{=} \lim_{t \to 0} \frac{5^t \cdot \ln 5 - 3^t \ln 3}{1} = \ln 5 - \ln 3 = \boxed{\ln \frac{5}{3}}$

$0/0$

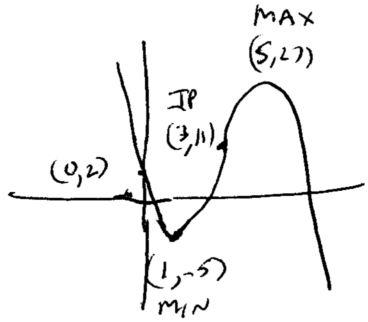
Sect 4.5

#3 $y = 2 - 15x + 9x^2 - x^3$

$y' = -15 + 18x - 3x^2 = -3(x^2 - 6x + 5)$
$= -3(x-5)(x-1)$

$y'' = 18x - 6x = 0 \Rightarrow \underline{x = 3}$

x	$f(x)$
1	-5
3	$+11$
5	$+27$



CD $(3, \infty)$
CU $(-\infty, 3)$

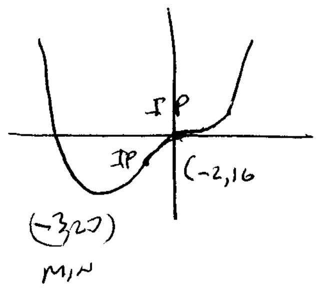
#5 $y = x^4 + 4x^3$

$y' = 4x^3 + 12x^2 = 4x^2(x+3) = 0$
$x = 0, \quad x = -3$

$y'' = 12x^2 + 24x = 12x(x+2) = 0 \quad x = -2$

$\underline{IP}$ at $\underline{x = 0}$ and $\underline{x = -2}$

x	$f(x)$
-3	-27
-2	-16
0	0



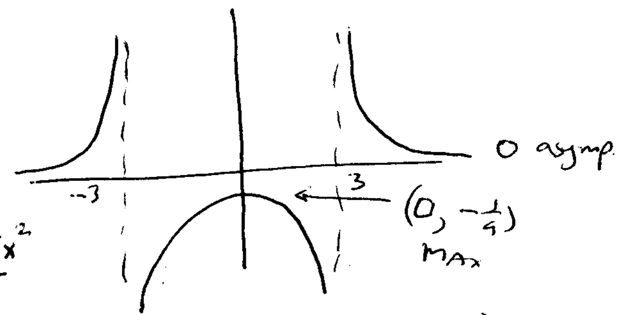
CU $(-\infty, -2)$
CD $(-2, 0)$
CU $(0, \infty)$

#11 $y = \frac{1}{x^2 - 9}$

$y' = \frac{d}{dx}(x^2 - 9)^{-1} = -1(x^2 - 9)^{-2} \cdot 2x = -\frac{2x}{(x^2-9)^2}$

$y'' = \frac{(x^2-9)^2(-2) - (-2x)2(x^2-9)(2x)}{(x^2-9)^4} = \frac{-2(x^2-9) + 8x^2}{(x^2-9)^3}$

$= \frac{6(x^2+3)}{(x^2-9)^3} \neq 0$



CU $(-\infty, -3)$ and $(3, \infty)$
CD $(-3, 3)$

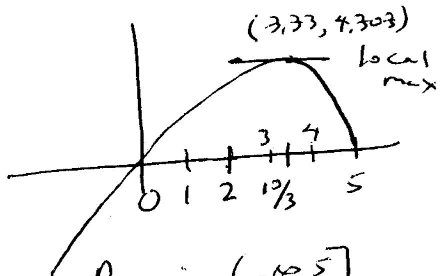
#19 $y = x\sqrt{5-x} = x(5-x)^{1/2}$

$y' = (5-x)^{1/2} + x\frac{1}{2}(5-x)^{-1/2}(-1) = \sqrt{5-x} - \frac{x}{2\sqrt{5-x}} = 0$

$\Rightarrow 2(5-x) = x \Rightarrow \underline{x = 10/3} \quad f(\frac{10}{3}) = 4.303$

$y'' = \frac{x-2}{4(5-x)^{3/2}} - \frac{1}{2\sqrt{5-x}} < 0 \quad \forall x$

CD everywhere



Domain $(-\infty, 5]$

Math 131 (4)

Prof. R.B. Goldstein

Chap 4 HW

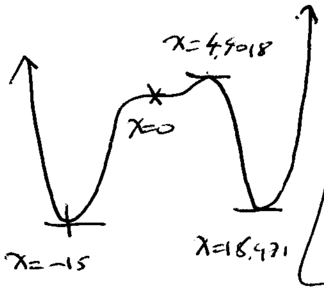
Stewart 6th Ed

Sect 4.6

#3 $f(x) = x^6 - 10x^5 - 400x^4 + 2500x^3$

$f'(x) = 6x^5 - 50x^4 - 1600x^3 + 7500x^2$

$f''(x) = 30x^4 - 200x^3 - 4800x^2 + 15000x$

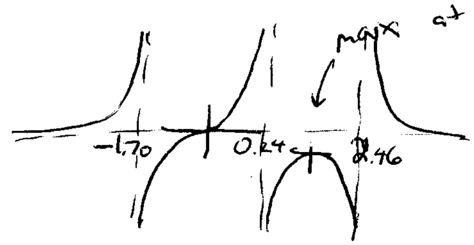


MIN IP MAX MIN

$f'(x)=0$ at $x = -15,\ 0,\ 4.4018,\ 18.931$

$f''(x)=0$ at $x = -11.337,\ 0,\ 2.925,\ 15.078 \Leftarrow$ IP.

#5 $f(x) = \dfrac{x}{x^3 - x^2 - 4x + 1}$



$f'(x) = \dfrac{+2x^3 - x^2 - 1}{(x^3 - x^2 - 4x + 1)^2} = 0$ at $x = 1$ only

from $f''(x) = 0$ $x = -0.424$ is an I.P.

Sect 4.7

#3 $\left.\begin{array}{l} xy = 100 \\ x + y \text{ is min} \end{array}\right\} \Rightarrow y = \dfrac{100}{x}$, $S = x + \dfrac{100}{x}$ is min

$S' = 1 - \dfrac{100}{x^2} = 0 \Rightarrow x = 10, -10$

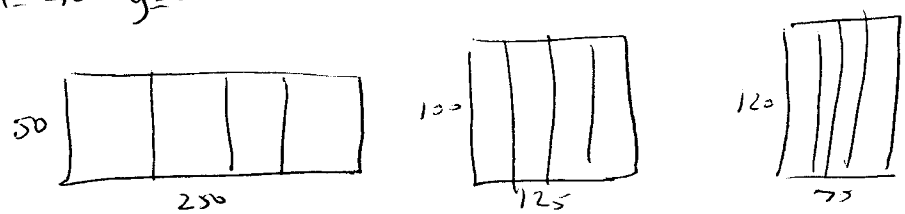
$S'' = \dfrac{200}{x^3}$ $S''(10) = \frac{1}{5}$ min $S''(-10) = -\frac{1}{5}$ max

if $x = 10$ $y = 10$ sum $= 20$

$x = -10$ $y = -10$ sum $= -20$

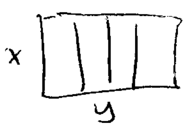
$\boxed{x = y = 10 \quad \text{Value} = 20}$

#9 (a)



A = 12,500 A = 12,500 A = 9000

(b)



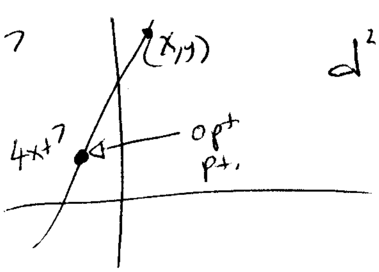
$A = xy$

(c) $L = 750 = 5x + 2y \Rightarrow y = 375 - \frac{5}{2}x$ $\Big\}$ $A = (375 - \frac{5}{2}x)x = 375x - \frac{5}{2}x^2$

$A' = 375 - 5x = 0 \Rightarrow x = 75$

$\underline{x = 75 \Rightarrow y = 187.5}$ $\underline{A = 14{,}062.5 \text{ ft}^2}$ a maximum

#17



$d^2 = x^2 + y^2 = x^2 + (4x+7)^2 = 17x^2 + 56x + 49$

$\frac{d}{dx}(d^2) = 34x + 56 = 0 \Rightarrow x = -\frac{28}{17}$

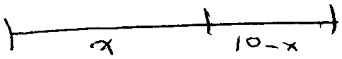
$4\left(-\frac{28}{17}\right) + 7 = \frac{7}{17}$ $y = \frac{7}{17}$

Prof. R.B. Goldstein

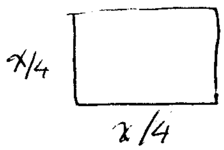
Chap 4 HW

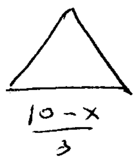
Stewart 6^{th} Ed

Sect 4.7



#33





$$A=\left(\frac{x}{4}\right)^2+\frac{1}{2}\left(\frac{10-x}{3}\right)\frac{\sqrt{3}}{2}\left(\frac{10-x}{3}\right)$$

$$=\frac{x^2}{16}+\frac{\sqrt{3}}{36}(10-x)^2 \quad \underline{0\le x\le 10}$$

$$A'(x)=\frac{x}{8}-\frac{\sqrt{3}}{18}(10-x)=0 \Rightarrow x=\frac{40\sqrt{3}}{9+4\sqrt{3}}\approx 4.35$$

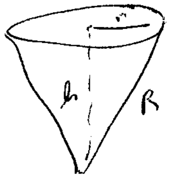
$$A(0)=\frac{\sqrt{3}}{36}100\cong 4.81$$

$$A(10)=\frac{100}{16}=6.25$$

max (all sq)

$$\boxed{A(4.35)\cong 2.72 \;\Uparrow\; \text{minimum}}$$

#37



$$h^2+r^2=R^2 \Rightarrow V=\frac{\pi}{3}r^2h=\frac{\pi}{3}(R^2-h^2)h=\frac{\pi}{3}(R^2h-h^3)$$

$$V'(h)=\frac{\pi}{3}(R^2-3h^2)=0 \text{ when } h=\frac{R}{\sqrt{3}}$$

$$V\left(\frac{R}{\sqrt{3}}\right)=\frac{2}{9\sqrt{3}}\pi R^3$$

#55 (a)

$$p(27000)=10 \qquad m=\frac{8-10}{33000-27000}=-\frac{1}{3000} \qquad y-10=-\frac{1}{3000}(x-27000)$$

$$p(33000)=8$$

$$y=p=-\frac{1}{3000}x+19$$

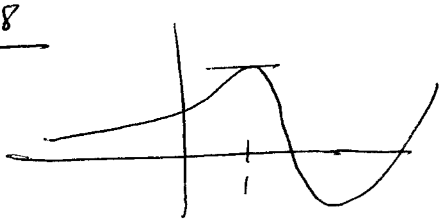
(b)

$$R=xp=19x-\frac{x^2}{3000} \qquad R'(x)=19-\frac{x}{1500}=0 \Rightarrow \underline{\underline{x=28{,}500}}$$

$$\underline{\underline{p=\$9.50}} \qquad R=\underline{\underline{\$270{,}750}}$$

Sect 4.8

#4



(a) $x_1=0$ fails ! $x_n\to-\infty$

(b) $x_1=1$ fails : 0 slope

(c) $x_1=3$ fails : goes neg. and $x_n\to-\infty$

(d) $x_1=4$ fails : 0 slope

(e) $x_1=5$ Success – finds a root

#5

$$x^3+2x-4=0 \quad x_1=1 \quad \Big\} \quad x_{n+1}=x_n-\frac{x_n^3+2x_n-4}{3x_n^2+2}$$

$$f'=3x^2+2$$

$$x_2=1-\frac{(-1)}{5}=1.2 \qquad x_3\cong 1.1797 \quad \text{(close to root } 1.179509\ldots)$$

#11

$$\sqrt[5]{20} \qquad x^5-20=0 \qquad x_{n+1}=x_n-\frac{x_n^5-20}{5x_n^4}$$

if $x_1=2 \Rightarrow x_2=1.85$

$x_3=1.82148$

$x_4=1.82056$

Sect 4.9

Math 131 (6)
Prof. R. B. Goldstein
Chap 4 HW
Stewart 6th Ed

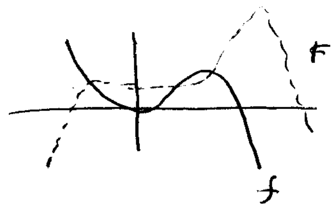
#3 $f(x) = \frac{1}{2} + \frac{3}{4}x^2 - \frac{4}{5}x^3 \Rightarrow \underline{\underline{F(x) = \frac{1}{2}x + \frac{1}{4}x^3 - \frac{1}{5}x^4 + c}}$

(note ck $F'(x) = f(x)$)

#13 $f(u) = \frac{u^4 + 3\sqrt{u}}{u^2} = u^2 + 3u^{-3/2} \Rightarrow \underline{\underline{F(u) = \frac{u^3}{3} - \frac{6}{\sqrt{u}} + c}}$

#21 $f(x) = 5x^4 - 2x^5 \quad F(0) = 4$

$\Rightarrow F(x) = x^5 - \frac{x^6}{3} + c \quad F(0) = C = 4$ } $F(x) = x^5 - \frac{x^6}{3} + 4$



$f(x) = 0$ when $F(x)$ has a local max

#40 $f''(t) = \frac{3}{\sqrt{t}} \quad f(4) = 20 \quad f'(4) = 7$

$f'(t) = 6t^{1/2} + c \qquad f'(4) = 12 + c = 7 \Rightarrow c = -5$

$f'(t) = 6t^{1/2} - 5 \qquad f(t) = 4t^{3/2} - 5t + D$ } $D = 8 \Rightarrow \boxed{f(t) = 4t^{3/2} - 5t + 8}$

$f(4) = 32 - 20 + D = 20$

#44 $f''(t) = 2e^t + 3\sin t \qquad f(0) = 0 \quad f(\pi) = 0$

$f'(t) = 2e^t - 3\cos t + c$

$f(t) = 2e^t - 3\sin t + ct + D$

$f(0) = 2 - 0 + D = 0 \Rightarrow \underline{D = -2}$

$f(\pi) = 2e^\pi - 0 + c\pi - 2 = 0 \Rightarrow c = \frac{2 - 2e^\pi}{\pi}$

$\therefore \underline{f(t) = 2e^t - 3\sin t + \left(\frac{2 - 2e^\pi}{\pi}\right)t - 2}$