Math 131 - Prof. Richard B. Goldstein - Chap 3 HW - Stewart 6e

Sect 3.1

#7 $\quad f(x) = x^3 - 4x + 6 \qquad f'(x) = 3x^2 - 4$

#17 $\quad G(x) = \sqrt{x} - 2e^x = x^{1/2} - 2e^x \qquad G'(x) = \frac{1}{2}x^{-1/2} - 2e^x = \frac{1}{2\sqrt{x}} - 2e^x$

#29 $\quad u = \sqrt[5]{t} + 4\sqrt{t^5} = t^{1/5} + 4t^{5/2} \qquad u' = \frac{1}{5}t^{-4/5} + 4\left(\frac{5}{2}\right)t^{3/2}$

$= \frac{1}{5t^{4/5}} + 10t^{3/2}$

#35 $\quad y = x^4 + 2e^x$ at $(0,2)$

$y' = 4x^3 + 2e^x \Big|_0 = 0 + 2 = 2$

(normal slope $= -\frac{1}{2}$)

$y - 2 = 2(x - 0)$

$y - 2 = 2x$

$y = 2x + 2$

$y - 2 = -\frac{1}{2}(x - 0)$

$y = -\frac{1}{2}x + 2$

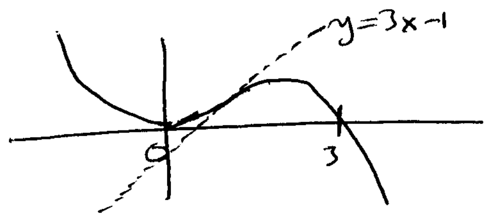
normal

#37 $\quad y = 3x^2 - x^3$ at $(1,2)$

$y' = 6x - 3x^2 \Big|_1 = 6 - 3 = 3$

$y - 2 = 3(x - 1)$

$y = 3x - 1$



Sect 3.2

#4 $\quad g(x) = \sqrt{x}\, e^x = x^{1/2} e^x \qquad g'(x) = \left(\frac{1}{2}x^{-1/2}\right)(e^x) + (x^{1/2})(e^x)$

$= \left(\frac{1}{2\sqrt{x}} + \sqrt{x}\right)e^x$

#14 $\quad y = \frac{x+1}{x^3 + x - 2} \qquad y' = \frac{(x^3 + x - 2)(x+1)' - (x+1)(x^3 + x - 2)'}{(x^3 + x - 2)^2}$

$= \frac{(x^3 + x - 2)(1) - (x+1)(3x^2 + 1)}{(x^3 + x - 2)^2} = \frac{-2x^3 - 3x^2 - 3}{(x^3 + x - 2)^2}$

#31 $\quad y = \frac{2x}{x+1}$ at $(1,1)$ $\qquad y' = \frac{(x+1)\cdot 2 - (2x)\cdot 1}{(x+1)^2} = \frac{2}{(x+1)^2}\Big|_{x=1} = \frac{1}{2}$

$y - 1 = \frac{1}{2}(x - 1)$ or $y = \frac{1}{2}x + \frac{1}{2}$

#32 $\quad y = \frac{e^x}{x}$ at $(1, e)$ $\qquad y' = \frac{x \cdot e^x - e^x \cdot 1}{x^2} = \frac{(x-1)e^x}{x^2}\Big|_{x=1} = 0$

$y - e = 0(x - 1) \Rightarrow y = e$ (horizontal line)

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Sect 3.2

#43 $f(5)=1 \quad f'(5)=6$
$g(5)=-3 \quad g'(5)=2$

(a) $(fg)'(5) = fg' + f'g\big|_{x=5} = 1\cdot 2 + 6\cdot(-3) = -16$

(b) $\left(\frac{f}{g}\right)'(5) = \frac{gf'-fg'}{g^2}\Big|_{x=5} = \frac{(-3)6 - 1(2)}{(-3)^2} = \frac{-20}{9}$

(c) $\left(\frac{g}{f}\right)'5 = \frac{fg'-gf'}{f^2}\Big|_{x=5} = \frac{(1)(2)-(-3)(6)}{1^2} = 20$

Sect 3.3

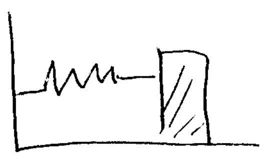
#5 $g(t) = t^3\cos t \qquad g'(t) = (3t^2)\cos t + t^3(-\sin t) = 3t^2\cos t - t^3\sin t$

#9 $y = \frac{x}{2-\tan x} \qquad y' = \frac{(2-\tan x)\cdot 1 - x(0-\sec^2 x)}{(2-\tan x)^2} = \frac{2-\tan x + x\sec^2 x}{(2-\tan x)^2}$

#11 $f(\theta) = \frac{\sec\theta}{1+\sec\theta} \qquad f'(\theta) = \frac{(1+\sec\theta)\cdot(\sec\theta\tan\theta) - \sec\theta(\sec\theta\tan\theta)}{(1+\sec\theta)^2}$

$= \frac{\sec\theta\tan\theta}{(1+\sec\theta)^2}$

#35



position $x = 8\sin t$ (cm)

(a) $\dot{x} = 8\cos t$
$\ddot{x} = -8\sin t$
$x(2\pi/3) = 8\sin(2\pi/3) = 8\sin 120° = 8\frac{\sqrt{3}}{2} = 4\sqrt{3}$ cm

(b) $\dot{x}\left(\frac{2\pi}{3}\right) = 8\cos\left(\frac{2\pi}{3}\right) = 8\cos(120°) = -4$ cm/sec

$\ddot{x}\left(\frac{2\pi}{3}\right) = -8\sin\left(\frac{2\pi}{3}\right) = -8\sin(120°) = -8\frac{\sqrt{3}}{2} = -4\sqrt{3}\ \frac{\text{cm}}{\text{sec}^2}$

Since $v = \dot{x} < 0$ it is moving to the left

Sect 3.4

#3 $y = (1-x^2)^{10} \qquad u = g(x) = 1-x^2 \quad y = f(u) = u^{10} \qquad y' = 10(1-x^2)^9\cdot(-2x)$
$= -20x(1-x^2)^9$

#11 $g(t) = \left(\frac{1}{t^4+1}\right)^3 = (t^4+1)^{-3} \qquad u = h(t) = t^4+1 \quad y = f(u) = u^{-3}$
$g'(t) = -3(t^4+1)^{-4}\cdot 4t^3 = \frac{-12t^3}{(t^4+1)^4}$

#25 $F(z) = \sqrt{\frac{z-1}{z+1}} = \left(\frac{z-1}{z+1}\right)^{1/2} \qquad F'(z) = \frac{1}{2}\left(\frac{z-1}{z+1}\right)^{-1/2}\cdot\left[\frac{(z+1)\cdot 1-(z-1)\cdot 1}{(z+1)^2}\right] = \frac{1}{2}\left(\frac{z-1}{z+1}\right)^{-1/2}\cdot\frac{2}{(z+1)^2}$

$\frac{1}{2}\,\frac{(z+1)^{1/2}}{(z-1)^{1/2}}\cdot\frac{2}{(z+1)^2} = \frac{1}{(z-1)^{1/2}(z+1)^{3/2}}$

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Sect 3.4

#29 $y = \sin(\tan 2x)$

$$y' = \cos(\tan 2x) \cdot (\tan 2x)'$$
$$= \cos(\tan 2x) \cdot \sec^2 2x \cdot 2$$
$$= 2\cos(\tan 2x)\sec^2 2x$$

Sect 3.5

#3 $\frac{1}{x} + \frac{1}{y} = 1$

$$-\frac{1}{x^2} - \frac{1}{y^2}y' = 0 \qquad y' = \frac{+\frac{1}{x^2}}{-\frac{1}{y^2}} = -\frac{y^2}{x^2}$$

$$\frac{1}{y} = 1 - \frac{1}{x}$$
$$y = \frac{1}{1-\frac{1}{x}} = \frac{x}{x-1} \qquad y' = \frac{(x-1)\cdot 1 - x(1)}{(x-1)^2} = -\frac{1}{(x-1)^2}$$

$$-\frac{y^2}{x^2} = -\frac{\left(\frac{x}{x-1}\right)^2}{x^2} = -\frac{x^2}{(x-1)x^2} = -\frac{1}{(x-1)^2}$$

$= \checkmark$

#8 $2x^3 + x^2y - xy^3 = 2$

$$6x^2 + x^2y' + 2xy - y^3 - x(3y^2y') = 0$$
$$(6x^2 + 2xy - y^3) + (x^2 - 3xy^2)y' = 0 \Rightarrow y' = \frac{6x^2 + 2xy - y^3}{3xy^2 - x^2}$$

#16 $\sqrt{x+y} = 1 + x^2y^2$

$$\tfrac{1}{2}(x+y)^{-1/2} \cdot (1 + y') = 0 + x^2(2yy') + 2xy^2$$
$$\frac{1}{2\sqrt{x+y}} + \frac{y'}{2\sqrt{x+y}} = (2x^2y)y' + 2xy^2$$
$$\left(\frac{1}{2\sqrt{x+y}} - 2x^2y\right)y' = 2xy^2 - \frac{1}{2\sqrt{x+y}}$$
$$y' = \frac{2xy^2 - \frac{1}{2\sqrt{x+y}}}{\frac{1}{2\sqrt{x+y}} - 2x^2y} = \boxed{\frac{4xy^2\sqrt{x+y} - 1}{1 - 4x^2y\sqrt{x+y}}}$$

#29 $2(x^2+y^2)^2 = 25(x^2-y^2)$ at 3,1

$$2(2)(x^2+y^2)\cdot(2x + 2yy') = 25(2x - 2yy')\Big|_{(3,1)} \Rightarrow 40(6 + 2y') = 25(6 - 2y')$$
$$240 + 80y' = 150 - 50y'$$
$$130y' = -90$$
$$y' = -9/13$$

$$y - 1 = -\tfrac{9}{13}(x-3)$$
$$y = -\tfrac{9}{13}x + \tfrac{40}{13}$$

#40 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{2x}{a^2} + \frac{2yy'}{b^2} = 0 \qquad y' = -\frac{2x/a^2}{2y/b^2} = -\frac{2xb^2}{2ya^2} = \frac{-xb^2}{ya^2}\Big|_{(x_0,y_0)} = \frac{-x_0b^2}{y_0a^2}$$

$$y - y_0 = -\frac{x_0b^2}{y_0a^2}(x - x_0)$$

multiply by $\frac{y_0}{b^2}$

$$\frac{y_0y}{b^2} - \frac{y_0^2}{b^2} = -\frac{x_0x}{a^2} + \frac{x_0^2}{a^2} \Rightarrow \boxed{\frac{x_0x}{a^2} + \frac{y_0y}{b^2} = \frac{x_0^2}{a^2} + \frac{y_0^2}{b^2} = 1}$$

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Sect 3.6

#4 $f(x) = \ln(\sin^2 x) \Rightarrow \quad f'(x) = \dfrac{1}{\sin^2 x} \cdot 2\sin x \cos x = \dfrac{2\cos x}{\sin x}$ or $2\cot x$

#5 $f(x) = \log_2(1-3x) = \dfrac{\ln(1-3x)}{\ln 2} \quad f'(x) = \dfrac{\frac{1}{1-3x}\cdot(-3)}{\ln 2} = \dfrac{-3}{(\ln 2)(1-3x)}$

#12 $h(x) = \ln(x + \sqrt{x^2-1}) \quad h'(x) = \dfrac{1}{x+\sqrt{x^2-1}} \cdot \left(1 + \frac{1}{2}(x^2-1)^{-1/2} \cdot 2x\right)$

$$= \frac{1 + \frac{x}{\sqrt{x^2-1}}}{x+\sqrt{x^2-1}} = \frac{\sqrt{x^2-1} + x}{(x+\sqrt{x^2-1})(\sqrt{x^2-1})} = \frac{1}{\sqrt{x^2-1}}$$

#25 $y'' = -\frac{1}{2} \cdot (x^2-1)^{-3/2}(2x) = \dfrac{-x}{(x^2-1)^{3/2}}$

#38 $y = \sqrt{x}\, e^{x^2} (x^2+1)^{10} \quad \ln y = \frac{1}{2}\ln x + x^2 + 10\ln(x^2+1)$

$$\frac{1}{y} y' = \frac{1}{2x} + 2x + 10\left(\frac{1}{x^2+1}\right)2x = \frac{1}{2x} + 2x + \frac{20x}{x^2+1}$$

$$\boxed{y' = \sqrt{x}\, e^{x^2}(x^2+1)^{10}\left[\frac{1}{2x} + 2x + \frac{20x}{x^2+1}\right]}$$

#43 $y = x^{\sin x}$

$\ln y = \sin x \ln x$

$\frac{1}{y} y' = \cos x \ln x + \sin x \left(\frac{1}{x}\right) = \cos x \ln x + \frac{\sin x}{x}$

$$y' = \boxed{x^{\sin x}\left(\cos x \ln x + \frac{\sin x}{x}\right)}$$

Sect 3.7

#1 $f(t) = t^3 - 12t^2 + 36t$

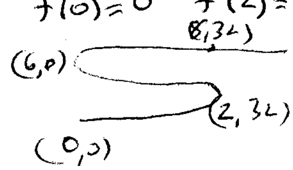
(a) $v = 3t^2 - 24t + 36$

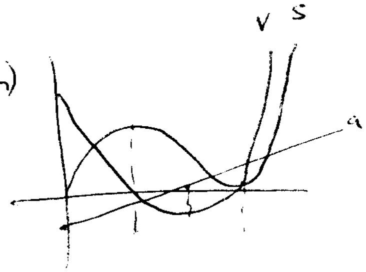
(b) $v(3) = 27 - 72 + 36 = -9$ ft/sec

(c) $v = 0 = 3(t^2 - 8t + 12) = 3(t-2)(t-6)$ at $t = 2, 6$ sec

(d) $v > 0$ when $t < 2$ or $t > 6$

(e) $f(0) = 0 \quad f(2) = 32 \quad f(6) = 0 \quad f(8) = 32$ total $|32| + |-32| + |32| = 96$ ft

(f) 

(h) 

(g) $a = 6t - 24 \quad a(3) = 18 - 24 = -6$ ft/sec²

(i) $\underline{v}$ and $\underline{a}$ same sign ≡ speeding up and when $2 < t < 4$ (both neg) $t > 6$ (both pos)

Sect 3.7

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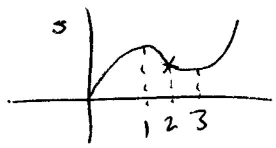
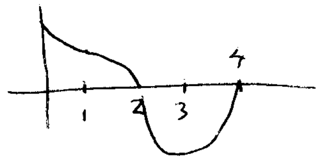
#6 (a) position



t	v	a
(0,1)	>0	<0
(1,2)	<0	<0 *
(2,3)	<0	>0
(3,4)	>0	>0 *

Speeding up from (1,2) and (3,4)

(b) position



t	v	a
(0,1)	<0	>0
(1,2)	<0	<0 *
(2,3)	<0	>0
(3,4)	>0	>0 *

Same as (a)

#20 $F = \frac{GmM}{r^2}$ $\frac{dF}{dr} = -\frac{2GmM}{r^3}$ force inward and decreasing as r increases

if $F' = 2 \frac{N}{Km}$ at $r = 20,000$ Km

then at $r = 10,000$ Km ($r^* = 0.5r$) we would have 8 times the force

or $16 \frac{N}{Km}$

Sect 3.8

#2 doubling time is 20 minutes, $P_0 = 60$

(a) $P(t) = 60e^{kt}$ t in hrs. $P(\frac{1}{3}) = 60e^{\frac{k}{3}} = 120$ $P(t) = 60e^{(\ln 8)t}$

$e^{k/3} = 2 \Rightarrow k = 3\ln 2 = \ln 8$

(b) $P(t) = 60(e^{\ln 8})^t = 60(8)^t$

(c) $P(8) = 60(8)^8 \approx 1,006,600,000$

(d) $\frac{dP}{dt} = kt$ $P'(8) = k\,P(8) = \ln 8\; P(8) \approx 2.093$ billion cells/hr

(e) $P(t) = 20000 = 60\, 8^t \Rightarrow 8^t = 1000/3$ $t \ln 8 = \ln\left(\frac{1000}{3}\right) \Rightarrow t = \frac{\ln(1000/3)}{\ln 8} \approx 2.79$ hr

#5 (a) Use 1750 $P(t) = P(1750)\, e^{k(t-1750)} \Rightarrow 980 = 790\, e^{k(1800-1750)}$

$\Rightarrow \frac{980}{790} = e^{50k} \Rightarrow k = \frac{1}{50}\ln\frac{980}{790} \approx 0.0043104$

$\therefore$ $P(1900) = 790\, e^{k(1900-1750)} \approx 1508$ million
$P(1950) = 790\, e^{k(1950-1750)} \approx 1871$ million } both are too low

(b) similarly using 1850 + 1900 $k = 0.005393$ gives $P(1950) = 2161$ million but closer

(c) now $k = 0.008785$ and $P(2000) = 3972$ which is also too low

assumption of constant rates of growth + death

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Sect 3.8

#11 ^{14}C has a half-life of 5,730 yrs.

$74\% \Rightarrow$ what age?

$y(t) = y(0)e^{-kt}$ $\quad y(5730) = \frac{y(0)}{2} = y(0)e^{-5730k}$

$e^{-5730k} = \frac{1}{2} \Rightarrow -5730k = \ln\frac{1}{2} \Rightarrow \boxed{k = \frac{\ln 2}{5730}}$

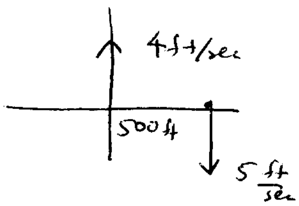
$y(t) = 0.74\, y(0) \Rightarrow 0.74\, e^{-\frac{t\ln 2}{5730}} \Rightarrow \ln 0.74 = -\frac{t\ln 2}{5730} \Rightarrow t = -\frac{5730\ln(0.74)}{\ln 2} \approx \boxed{2489\text{ yrs}}$

Sect 3.9

#7 $\left.\begin{array}{l} y = x^3 + 2x \\ x = 2,\ \dot{x} = 5 \end{array}\right\} \Rightarrow \dot{y} = 3x^2\dot{x} + 2\dot{x} = (3x^2+2)\dot{x}$

$= [3(2)^2+2]\,5 = 14 \cdot 5 = \boxed{70}$

#17



$(x_1, y_1) = (0, 4t)$

$(x_2, y_2) = (500, -300 - 5t)$ $\quad t = 20 \text{ mins} = 1200 \text{ sec}$

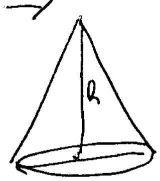
$d^2 = (0-500)^2 + (-300-9t)^2$ $\quad d^2 = 500^2 + 11100^2 = 123{,}460{,}000$

$d = 11111.255$

$\cancel{2}d\dot{d} = \cancel{2}(-300-9t)\cdot(-9)$

$\dot{d} = \frac{(-11{,}100)(-9)}{11{,}111.255} = 8.99 \text{ ft/sec}$

#27 $\Rightarrow$



$b = 2r = h$ $\quad \dot{V} = 30\ \frac{ft^3}{min}$

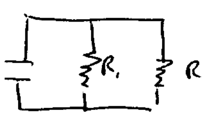
$h = 10$ ft

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{12}$

$\dot{V} = \frac{3\pi h^2}{12}\dot{h} = \frac{\pi h^2}{4}\dot{h}$

$\dot{h} = \frac{30 \cdot 4}{\pi(10)^2} = \boxed{\frac{6}{5\pi} \approx 0.38\ \frac{ft}{min}}$

#33 $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$



$\dot{R}_1 = 0.3\ \frac{\Omega}{s}$ $\quad R_1 = 80\ \Omega$

$\dot{R}_2 = 0.2\ \frac{\Omega}{s}$ $\quad R_2 = 100\ \Omega$

$\frac{1}{R} = \frac{1}{80} + \frac{1}{100} = .0225$

$R = 44.44\ldots\ \Omega$

$-\frac{1}{R^2}\dot{R} = -\frac{1}{R_1^2}\dot{R}_1 - \frac{1}{R_2^2}\dot{R}_2$ $\quad -\frac{1}{(44.44\ldots)^2}\dot{R} = -\frac{0.3}{80^2} - \frac{0.2}{100^2} = -6.6875\times10^{-5} \Rightarrow \boxed{\dot{R} = 0.132\ \frac{\Omega}{s}}$

Sect 3.10

#3 $f(x) = \cos x$ $\quad a = \frac{\pi}{2}$ $\quad f\left(\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0$

$f'(x) = -\sin x$ $\quad f'\left(\frac{\pi}{2}\right) = -\sin\left(\frac{\pi}{2}\right) = -1$

$f(a) + f'(a)(x-a)$

$\boxed{0 + (-1)\left(x - \frac{\pi}{2}\right) = \frac{\pi}{2} - x}$

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Sect 3.10

#7 $\sqrt[3]{1-x} \approx 1 - \frac{1}{3}x$

$f(x) = (1-x)^{1/3}$ $f(0) = 1^{1/3} = 1$

$f'(x) = \frac{1}{3}(1-x)^{-2/3}(-1)$ $f'(0) = -\frac{1}{3}$

$\therefore 1 - \frac{1}{3}(x-0) = 1 - \frac{1}{3}x$

When is $\sqrt[3]{1-x} - 0.1 < 1 - \frac{1}{3}x < \sqrt[3]{1-x} + 0.1$?

* using numerical/graphical methods: $-1.204 < x < 0.706$

#25 $(8.06)^{2/3}$ $f(x) = (8+x)^{2/3}$ $f(0) = 8^{2/3} = 4$

$f'(x) = \frac{2}{3}(8+x)^{-1/3}$ $f'(0) = \frac{2}{3}(8)^{-1/3} = \frac{2}{3 \cdot 2} = \frac{1}{3}$

$\Big\} \; 4 + \frac{1}{3}(x-0)$

$8.06^{2/3} \approx 4 + \frac{1}{3}(.06) = \boxed{4.02}$

Sect 3.11

#3 (a) $\sinh(\ln 2) = \frac{e^{\ln 2} - e^{-\ln 2}}{2} = \frac{2 - \frac{1}{2}}{2} = 0.75$

(b) $\sinh(2) = \frac{e^2 - e^{-2}}{2} \sim 3.627$

#11 $\sinh(x+y) = \frac{e^{x+y} - e^{-x-y}}{2}$

$\sinh x \cosh y + \cosh x \sinh y = \left(\frac{e^x - e^{-x}}{2}\right)\left(\frac{e^y + e^{-y}}{2}\right) + \left(\frac{e^x + e^{-x}}{2}\right)\left(\frac{e^y - e^{-y}}{2}\right)$

$= \frac{e^{x+y} + \cancel{e^{x-y}} - \cancel{e^{-x+y}} - e^{-x-y} + e^{x+y} - \cancel{e^{x-y}} + \cancel{e^{-x+y}} - e^{-x-y}}{4}$

$= \frac{2e^{x+y} - 2e^{-x-y}}{4} = \frac{e^{x+y} - e^{-x-y}}{2} = \sinh(x+y)$ ✓

#20 $\tanh x = \frac{12}{13} \Rightarrow \coth x = \frac{1}{\tanh x} = \frac{1}{12/13} = \frac{13}{12}$

$\text{sech}^2 x = 1 - \tanh^2 x = 1 - \frac{12^2}{13^2} = \frac{25}{169} \Rightarrow \text{sech}\, x = \frac{5}{13}$

$\cosh x = \frac{1}{\text{sech}\, x} = \frac{13}{5}$ $\tanh x = \frac{\sinh x}{\cosh x} \Rightarrow \left(\frac{12}{13}\right) = \frac{\sinh x}{13/5} \Rightarrow \sinh x = \frac{12}{5}$

$\text{csch}\, x = \frac{1}{\sinh x} = \frac{5}{12}$

#32 $g(x) = \cosh(\ln x)$

$g'(x) = \sinh(\ln x) \cdot \frac{1}{x} = \frac{e^{\ln x} - e^{-\ln x}}{2} \cdot \frac{1}{x} = \frac{x - \frac{1}{x}}{2} \cdot \frac{1}{x} = \frac{x - \frac{1}{x}}{2x} = \frac{x^2 - 1}{2x^2}$

$= \frac{1}{2} - \frac{1}{2x^2}$