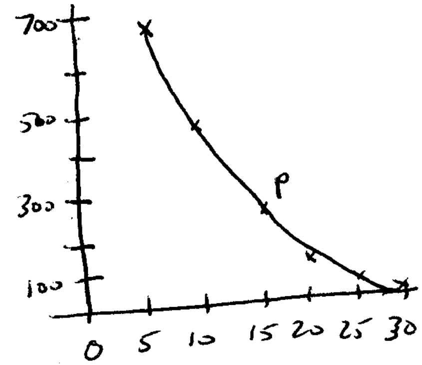
Math 131 - Prof. Richard B. Goldstein - Stewart 6^E ed - Chap 2 HW

Sect 2.1

#1

| t (min) | 5 | 10 | 15 (pt. P) | 20 | 25 | 30 |
|---|---|---|---|---|---|---|
| V (gal) | 694 | 444 | 250 | 111 | 28 | 0 |



(a) Q : (5,694) $m_{PQ} = \frac{694-250}{5-15} = -\frac{444}{10} = -44.4$

Q : (10,444) $m_{PQ} = \frac{444-250}{10-15} = -\frac{194}{5} = -38.8$

Q : (20,111) $m_{PQ} = \frac{111-250}{20-15} = -\frac{139}{5} = -27.8$

Q : (25,28) $m_{PQ} = \frac{28-250}{25-15} = -\frac{222}{10} = -22.2$

Q : (30,0) $m_{PQ} = \frac{0-250}{30-15} = -\frac{250}{15} = -16.\overline{6}$

(b) two closest pts $t=10, 20$ $\frac{(-38.8)+(-27.8)}{2} = -33.3$

#3 $y = \frac{x}{1+x}$ pt. $P(1, \frac{1}{2})$

| x | 0.5 | 0.9 | 0.99 | 0.999 |
|---|---|---|---|---|
| Q | $(0.5, 0.\overline{3})$ | (0.9, 0.473) | (0.99, 0.4975) | (0.999, 0.49975) |
| $m_{PQ}$ | $0.3\overline{3}$ | 0.263 | 0.2513 | 0.250125 |

$m \to 0.25$

$y = \frac{1}{4}x + \frac{1}{4}$

similarly for $x = 1.5$ $m_{PQ} = 0.2$, $x = 1.1$ $m_{PQ} = 0.238$, $x = 1.001$ $m_{PQ} = 0.24987$

#5 $y(t) = 40t - 16t^2$ $y(2) = 40(2) - 16(2)^2 = 16$

(a) $V_{avg} = \frac{y(2+h) - y(2)}{2+h-2} = \frac{[40(2+h) - 16(2+h)^2] - 16}{h} = \frac{-24h - 16h^2}{h} = -24 - 16h$ if $h \neq 0$

(i) $[2, 2.5]$ $h = 0.5$ $V_{avg} = -32$ ft/sec

(ii) $[2, 2.1]$ $h = 0.1$ $V_{avg} = -25.6$ ft/sec

(iii) $[2, 2.05]$ $h = 0.05$ $V_{avg} = -24.8$ ft/sec

(iv) $[2, 2.01]$ $h = 0.01$ $V_{avg} = -24.16$ ft/sec

(b) as $t \to 2$

$-24$ ft/sec

Sect 2.2

#1 $\lim_{x \to 2} f(x) = 5$ as $x$ approaches 2 $f(x)$ approaches 5

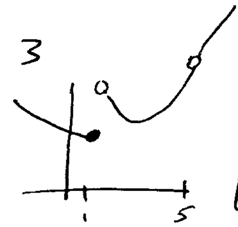
$f(2)$ could be any value ($f(2) = 3$ is OK)

#3 (a) $\lim_{x \to -3} f(x) = \infty$ $f(x)$ can be as large (arbitrarily) as we like by taking $x$ closer to $-3$ (but not equal to $-3$)

(b) $\lim_{x \to 4^+} f(x) = -\infty$ $f(x)$ can be as large negatively as we like by taking $x$ closer to 4 but always being above 4

Sect 2.2

#5 (a) $\lim_{x\to 1^-} f(x)=2$  (b) $\lim_{x\to 1^+} f(x)=3$



(c) $\lim_{x\to 1} f(x)$ D.N.E.  $2\neq 3$

(d) $\lim_{x\to 5} f(x)=4$  (e) $f(5)$ D.N.E.

# 18 $f(x)=\dfrac{x^2-2x}{x^2-x-2} = \dfrac{(x-2)x}{(x-2)(x+1)} = \dfrac{x}{x+1}$ if $x\neq 2$

| $x$ | 2.5 | 2.1 | 2.05 | 2.01 | 2.005 | 2.001 | |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 0.714... | 0.677... | 0.672 | 0.667... | 0.6672... | 0.666778 | $\to 2/3$ |

| $x$ | 1.9 | 1.95 | 1.99 | 1.995 | 1.999 | |
|---|---|---|---|---|---|---|
| $f(x)$ | 0.655... | 0.661... | 0.6655... | 0.6661... | 0.666556... | $\to 2/3$ |

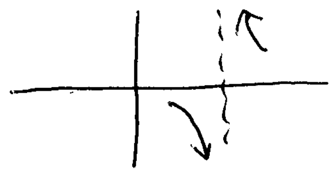
#21 $f(x)=\dfrac{\sqrt{x+4}-2}{x} = \dfrac{\sqrt{x+4}-2}{x}\cdot\dfrac{\sqrt{x+4}+2}{\sqrt{x+4}+2} = \dfrac{x+4-4}{x(\sqrt{x+4}+2)} = \dfrac{1}{\sqrt{x+4}+2}$ if $x\neq 0$

| $x$ | 1 | 0.5 | 0.1 | 0.05 | 0.01 | |
|---|---|---|---|---|---|---|
| $f(x)$ | 0.236... | 0.2426... | 0.2484... | 0.2492... | 0.2498... | $\to 1/4$ |

| $x$ | −1 | −0.5 | −0.1 | −0.05 | −0.01 | |
|---|---|---|---|---|---|---|
| $f(x)$ | 0.267... | 0.258... | 0.2515... | 0.2507... | 0.2501... | $\to 1/4$ |

#33

$\lim_{x\to 1^-} \dfrac{1}{x^3-1} = -\infty$  $\lim_{x\to 1^+} \dfrac{1}{x^3-1} = +\infty$



for example $f(0.99) = -33.7$  for ex. $f(1.01) = +33.0$

infinite / no limit exists

Sect 2.3

#3 $\lim_{x\to -2} (3x^4+2x^2-x+1) = \lim_{x\to -2} 3x^4 + \lim_{x\to -2} 2x^2 - \lim_{x\to -2} x + \lim_{x\to -2} 1$  (laws #1,2)

$= 3\lim_{x\to -2} x^4 + 2\lim_{x\to -2} x^2 - \lim_{x\to -2} x + \lim_{x\to -2} 1$  (#3)

$= 3(-2)^4 + 2(-2)^2 - (-2) + 1 = 48+8+2+1 = 59$
(#7,8,9)

#9 $\lim_{x\to 4^-} \sqrt{16-x^2} = \sqrt{\lim_{x\to 4^-}(16-x^2)} = \sqrt{16-(4)^2} = \sqrt{0} = 0$

rule #11  rule 2,7,9

#10 (a) $\dfrac{x^2+x-6}{x-2} = x+3$ is not true at $x=2$ only (left side undefined)

(b) Since they are the same for all but $x=2$ limit is the same (replacement theorem)

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## Sect 2.3

#15 $\lim_{t\to -3} \frac{t^2-9}{2t^2+7t+3}$ since $t^2-9\Big|_{t=-3}=0$ and $2t^2+7t+3\Big|_{t=-3}=0$

$t+3$ is a common factor

$$\lim_{t\to -3}\frac{(t+3)(t-3)}{(t+3)(2t+1)} = \lim_{t\to -3}\frac{t-3}{2t+1} = \frac{-6}{-5} = 1.2$$ by replacement theorem

#29 $$\lim_{t\to 0}\left(\frac{1}{t\sqrt{1+t}}-\frac{1}{t}\right) = \lim_{t\to 0}\left(\frac{1-\sqrt{1+t}}{t\sqrt{1+t}}\right) = \lim_{t\to 0}\left(\frac{1-\sqrt{1+t}}{t\sqrt{1+t}}\cdot\frac{1+\sqrt{1+t}}{1+\sqrt{1+t}}\right)$$
$\infty - \infty$

$$= \lim_{t\to 0}\left(\frac{1-(1+t)}{t\sqrt{1+t}\,(1+\sqrt{1+t})}\right) = \lim_{t\to 0}\left(\frac{-t}{t\sqrt{1+t}\,(1+\sqrt{1+t})}\right) \overset{\text{by repl. th.}}{\Longrightarrow} \lim_{t\to 0}\left(\frac{-1}{\sqrt{1+t}\,(1+\sqrt{1+t})}\right)$$

$$= \lim_{t\to 0}\left(\frac{-1}{1(1+1)}\right) = -\frac{1}{2}$$

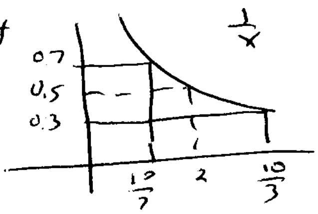
#61 $\lim_{x\to -2}\frac{3x^2+ax+a+3}{x^2+x-2}$ since $x^2+x-2\Big|_{-2}=0$ $x+2$ must be a common factor / i.e. $x=-2$ is a root

$$3x^2+ax+a+3\Big|_{-2} = 12-2a+a+3 = 15-a = 0 \Rightarrow a=15$$

$$\lim_{x\to -2}\frac{3x^2+15x+18}{x^2+x-2} = \lim_{x\to -2}\frac{(x+2)(3x+9)}{(x+2)(x-1)} \underset{\text{repl. th.}}{=} \lim_{x\to -2}\frac{3x+9}{x-1} = \frac{+3}{-3} = -1$$

## Sect 2.4

#1



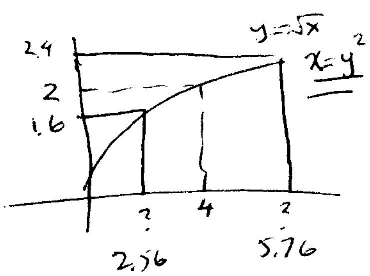

$|x-2| < \delta \Rightarrow \left|\frac{1}{x}-0.5\right| < 0.2$

On the left side $|x-2| < \left|\frac{10}{7}-2\right| = \frac{4}{7}$

On the right side $|x-2| < \left|\frac{10}{3}-2\right| = \frac{4}{3}$

if $\delta = \min\left(\frac{4}{7},\frac{4}{3}\right) = \frac{4}{7}$, both will be true

#3




$\sqrt{x}=1.6 \Rightarrow x=2.56$

$\sqrt{x}=2.4 \Rightarrow x=5.76$

$|x-4| < \delta \Rightarrow |\sqrt{x}-2| < 0.4$

On the left side $|x-4| < |2.56-4| = 1.44$

On the right side $|x-4| < |5.76-4| = 1.76$

if $\delta = \min(1.44, 1.76) = 1.44$, both will be true

**Sect 2.4**

given an $\varepsilon > 0$ we need to find a $\delta > 0$

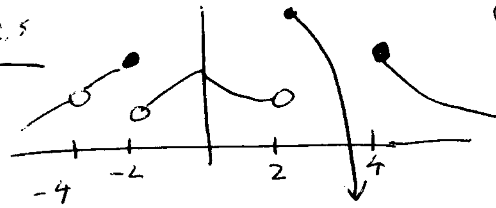
s.t. $0 < |x-2| < \delta \Rightarrow \left| \dfrac{x^2+x-6}{x-2} - 5 \right| < \varepsilon$

Since $\dfrac{x^2+x-6}{x-2} = x+3$ when $x \neq 2$ we need

$|x+3-5| < \varepsilon$ or $|x-2| < \varepsilon$ $\therefore$ choose $\underline{\underline{\delta = \varepsilon}}$

**Sect 2.5**

**#3**




(a)

| discontinuity | type |
|---|---|
| $-4$ | removable |
| $-2$ | jump |
| $+2$ | jump |
| $+4$ | infinite |

(b) $f$ is continuous from the left at $x=-2$ since $\lim_{x \to -2^-} f(x) = f(-2)$

$f$ is continuous from the right at $x=2$ and $x=4$

$f$ is discontinuous at $x=-4$ from either side since $f(-4)$ is undef.

**#13** $f(x) = \dfrac{2x+3}{x-2}$ $(2, \infty)$

for $\underline{a > 2}$ $\lim_{x \to a} \dfrac{2x+3}{x-2} = \dfrac{2a+3}{a-2} = f(a)$

**#17** $f(x) = \begin{cases} e^x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$ $a = 0$ $\overset{\text{left}}{\lim_{x \to 0^-} e^x} = 1$ $\overset{\text{right}}{\lim_{x \to 0^+} x^2} = 0$

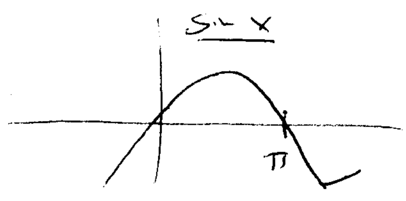
Since $0 \neq 1$ $\lim_{x \to 0} f(x)$ D.N.E. $\therefore$ <u>not continuous</u>

**#43** (a) $f(x) = \dfrac{x^4-1}{x-1}$, $a=1$ $\lim_{x \to 1} \dfrac{x^4-1}{x-1} = \lim_{x \to 1} \underbrace{(x+1)(x^2+1)}_{g(x)} = 4$ <u>removable</u>

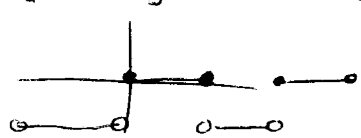
(b) $f(x) = \dfrac{x^3-x^2-2x}{x-2}$, $a=2$ $\lim_{x \to 2} \dfrac{x^3-x^2-2x}{x-2} = \lim_{x \to 2} \underbrace{x^2+x}_{g(x)} = 6$ <u>removable</u>

(c) $f(x) = [\![ \sin x ]\!]$, $a = \pi$ $\lim_{x \to \pi^-} [\![ \sin x ]\!] = 0$

$\underline{\sin x}$




$[\![ \sin x ]\!]$



but $\lim_{x \to \pi^+} [\![ \sin x ]\!] = -1$

$\lim_{x \to \pi} [\![ \sin x ]\!]$ D.N.E.

<u>jump discontinuity</u>

Sect 2.6

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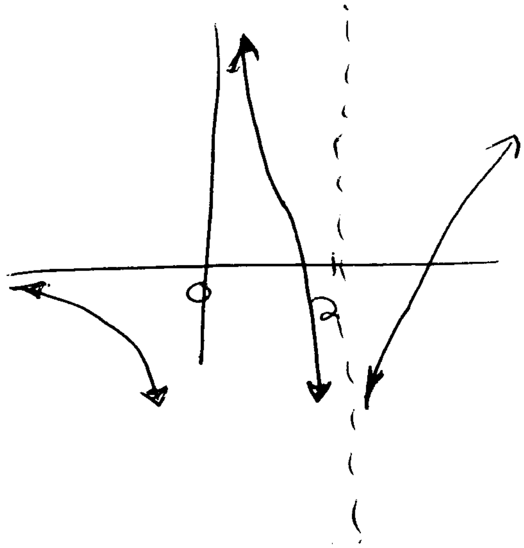
#7 $\lim_{x\to 2} f(x) = -\infty$

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to -\infty} f(x) = 0$

$\lim_{x\to 0^+} f(x) = \infty$

$\lim_{x\to 0^-} f(x) = -\infty$

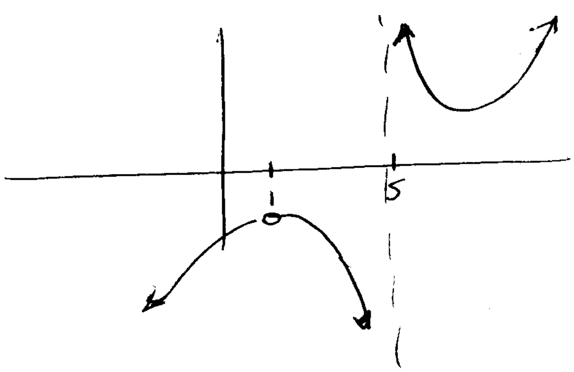



#14 $\lim_{x\to\infty} \sqrt{\dfrac{12x^3-5x+2}{1+4x^2+3x^3}} \underset{\text{Th. 5}}{\Longrightarrow} \lim_{x\to\infty} \sqrt{\dfrac{12-\frac{5}{x^2}+\frac{2}{x^3}}{\frac{1}{x^3}+\frac{4}{x}+3}} = \lim_{x\to\infty} \sqrt{\frac{12}{3}} = \sqrt{4} = 2$

#25 $\lim_{x\to\infty} (\sqrt{9x^2+x} - 3x)$ $\infty - \infty$ $= \lim_{x\to\infty} \left(\sqrt{9x^2+x} - 3x \cdot \dfrac{\sqrt{9x^2+x}+3x}{\sqrt{9x^2+x}+3x}\right) = \lim_{x\to\infty} \left(\dfrac{9x^2+x-9x^2}{\sqrt{9x^2+x}+3x}\right)$

$= \lim_{x\to\infty} \left(\dfrac{x}{\sqrt{9x^2+x}+3x}\right) = \lim_{x\to\infty} \left(\dfrac{1}{\sqrt{9+\frac{1}{x}}+3}\right) = \lim_{x\to\infty} \left(\dfrac{1}{3+3}\right) = \dfrac{1}{6}$

#43 $y = \dfrac{x^3-x}{x^2-6x+5} = \dfrac{x(x-1)(x+1)}{(x-5)(x-1)} = \begin{cases} \dfrac{x(x+1)}{x-5} & \text{if } x \neq 1 \\ \text{DNE} & \text{if } x = 1 \end{cases}$ $\leftarrow g(x)$ note $g(1) = -\frac{1}{2}$




no horizontal asymptotes

Vertical asymptote at $x=5$

Sect 2.7

#6 $y = 2x^3 - 5x$ $(-1, 3)$ $m = \lim_{x\to -1} \dfrac{(2x^3-5x)-3}{x-(-1)} = \lim_{x\to -1} \dfrac{2x^3-5x-3}{x+1} = \lim_{x\to -1} (2x^2-2x-3)$
$= 1$

$y - 3 = 1(x-(-1)) \Rightarrow y = x+4$

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## Sect 2.7

#7 $y=\sqrt{x}$, $(1,1)$ $m=\lim\limits_{x\to 1}\dfrac{\sqrt{x}-1}{x-1}=\lim\limits_{x\to 1}\dfrac{(\sqrt{x}-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x}+1)}$

$=\lim\limits_{x\to 1}\dfrac{x-1}{(x-1)(\sqrt{x}+1)}=\lim\limits_{x\to 1}\dfrac{1}{\sqrt{x}+1}=\dfrac{1}{2}$

tangent line $y-1=\frac{1}{2}(x-1)$ or $y=\frac{1}{2}x+\frac{1}{2}$

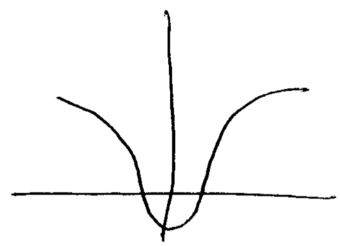
#15 $v(a)=\lim\limits_{h\to 0}\dfrac{s(a+h)-s(a)}{h}=\lim\limits_{h\to 0}\dfrac{\frac{1}{(a+h)^2}-\frac{1}{a^2}}{h}=\lim\limits_{h\to 0}\dfrac{\frac{a^2-(a+h)^2}{a^2(a+h)^2}}{h}$

$=\lim\limits_{h\to 0}\dfrac{a^2-(a^2+2ah+h^2)}{ha^2(a+h)^2}=\lim\limits_{h\to 0}\dfrac{-2ah-h^2}{ha^2(a+h)^2}=\lim\limits_{h\to 0}\dfrac{-2a-h}{a^2(a+h)^2}=-\dfrac{2a}{a^2a^2}=\dfrac{-2}{a^3}$

$v(1)=-\frac{2}{1^3}=-2\ \frac{m}{s}$ $\quad v(2)=-\frac{2}{2^3}=-\frac{1}{4}\ \frac{m}{s}$ and $v(3)=-\frac{2}{3^3}=-\frac{2}{27}\ \frac{m}{s}$

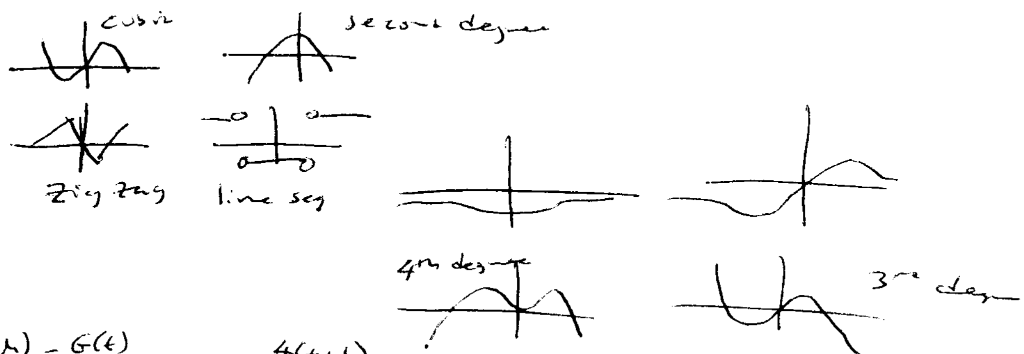
## Sect 2.8

#1 $f$ is an odd function $\Rightarrow$ $f'$ is an even function



(a) $f'(-3)\sim 1.5$ (d) $f'(0)\sim -4 \leftarrow$ min
(b) $f'(-2)\sim 1$ (e) $f'(1)\sim 0$ (g) $f'(3)\sim 1.5$
(c) $f'(-1)\sim 0$ (f) $f'(2)\sim 1$

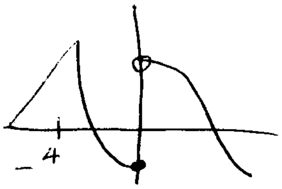
#3 (a) $\Leftrightarrow$ II
(b) $\Leftrightarrow$ IV
(c) $\Leftrightarrow$ I
(d) $\Leftrightarrow$ III




#27 $G'(t)=\lim\limits_{h\to 0}\dfrac{G(t+h)-G(t)}{h}=\lim\limits_{h\to 0}\dfrac{\frac{4(t+h)}{(t+h)+1}-\frac{4t}{t+1}}{h}=\lim\limits_{h\to 0}\dfrac{4(t+h)(t+1)-4t(t+h+1)}{h(t+h+1)(t+1)}$

$=\lim\limits_{h\to 0}\dfrac{4h}{h(t+h+1)(t+1)}=\dfrac{4}{(t+1)^2}$ domain of $G$ = domain of $G'$
$(-\infty,-1)\cup(-1,\infty)$

#35




$f$ is not differentiable at $x=-4$
since there is a cusp/corner there
nor at $x=0$ since there is a jump discontinuity there