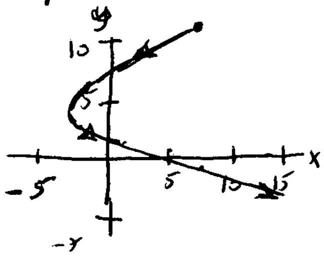
Math 132 - Prof. Richard B. Goldstein - Stewart 6^{ed} ET - Chapter 10

Sect 10.1

#7 $\quad x = t^2 - 2$, $\quad y = 5 - 2t$ $\qquad -3 \le t \le 4$

t	-3	-2	-1	0	1	2	3	4
x	7	2	-1	-2	-1	2	7	14
y	11	9	7	5	3	1	-1	-3



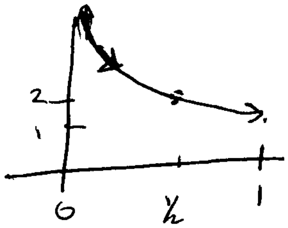
$$t = \frac{5-y}{2} \Rightarrow x = \left(\frac{5-y}{2}\right)^2 - 2 \Rightarrow \boxed{x = \frac{y^2}{4} - \frac{5}{2}y + \frac{9}{4}}$$

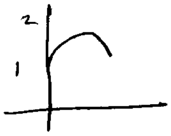
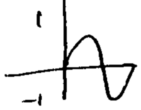
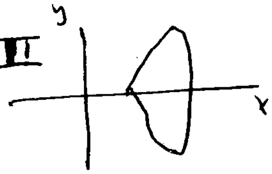
#13 $\quad x = \sin t$ $\qquad 0 < t < \frac{\pi}{2}$

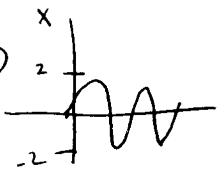
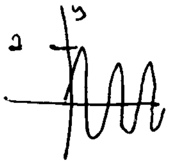
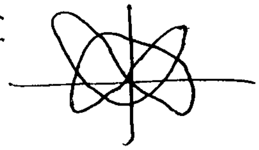
$y = \csc t = \frac{1}{\sin t}$

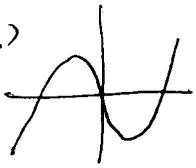
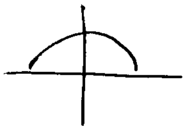
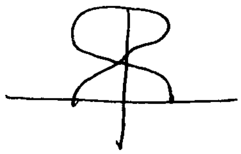
$\boxed{y = \frac{1}{x}}$

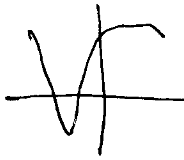
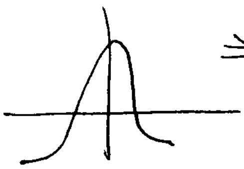
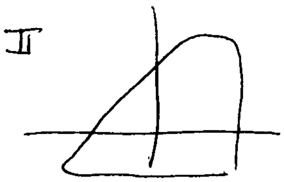
t	0	$\pi/6$	$\pi/3$	$\pi/2$
x	0	1/2	$\sqrt{3}/2$	1
y	∞	2	$2/\sqrt{3}$	1



#24 (a)   $\Rightarrow$ III 

(b)   $\Rightarrow$ I 

(c)   $\Rightarrow$ IV 

(d)   $\Rightarrow$ II 

Sect 10.2

#5 $\quad x = e^{\sqrt{t}}$, $\quad y = t - \ln t^2 = t - 2\ln t$ $\quad \}$ at $t = 1$

$$\dot{x} = e^{\sqrt{t}} \cdot \frac{1}{2} t^{-1/2}\Big|_{t=1} = e \cdot \frac{1}{2} \cdot 1 = \frac{e}{2}$$

$$\dot{y} = 1 - \frac{2}{t}\Big|_{t=1} = 1 - 2 = -1$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-1}{e/2} = \boxed{-\frac{2}{e}}$$

at $t = 1$ $(x, y) = (e, 1)$ $\quad \boxed{y - 1 = -\frac{2}{e}(x - e)}$

#11 $\quad x = 4 + t^2$, $\quad y = t^2 + t^3$

$$\frac{dx}{dt} = 2t \quad \frac{dy}{dt} = 2t + 3t^2 \Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t + 3t^2}{2t} = \boxed{1 + \frac{3}{2}t}$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{3}{2}}{2t} = \boxed{\frac{3}{4t}}$$

Concave up for $\boxed{t > 0}$

Sect 10.2

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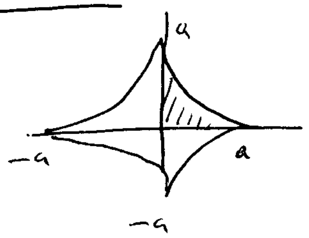
#34



$x = a\cos^3\theta = f(\theta)$

$y = a\sin^3\theta = g(\theta)$

$A = \int_0^a y\,dx$

$\int_\alpha^\beta g(\theta)\, f'(\theta)\,d\theta \Rightarrow 4\int_0^{\pi/2} (a\sin^3\theta)(3a\cos^2\theta \cdot -\sin\theta)\,d\theta$

$A = -12a^2\int_0^{\pi/2} \sin^4\theta\cos^2\theta \overset{\text{Using Derive}}{=} 12a^2\left[\frac{\theta}{16} - \frac{\sin 4\theta}{16} - \frac{\sin^3 2\theta}{48}\right]_0^{\pi/2} = 12a^2\left(\frac{\pi}{32}\right) = \boxed{\frac{3\pi a^2}{8}}$

#41

$x = 1 + 3t^2$ $\qquad 0 \le t \le 1$ $\qquad \dot{x} = 6t$

$y = 4 + 2t^3$ $\qquad \dot{y} = 6t^2$

$L = \int_0^1 \sqrt{(6t)^2 + (6t^2)^2}\,dt = \int_0^1 6t\sqrt{1+t^2}\,dt \xrightarrow[du = 2t\,dt]{u = 1+t^2} \int_1^2 3u^{1/2}\,du = 2u^{3/2}\Big|_1^2$

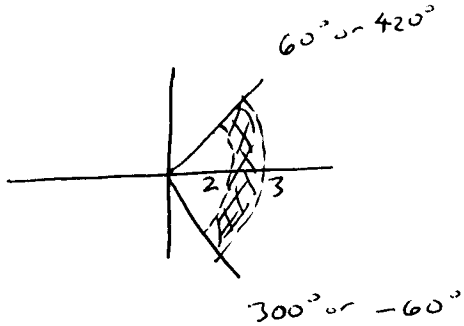
$= 2(2\sqrt{2} - 1) = \boxed{4\sqrt{2} - 2}$

Sect 10.3

#11 $\quad 2 < r < 3$

$\frac{5\pi}{3} \le \theta \le \frac{7\pi}{3}$

$\quad 300^\circ \qquad 420^\circ$ or 60°



#25 $\quad x^2 + y^2 = 2cx$

$x = r\cos\theta$

$y = r\sin\theta$

$r^2\cos^2\theta + r^2\sin^2\theta = 2cr\cos\theta$

$r^2 = 2rc\cos\theta$

$\boxed{r = 2c\cos\theta}$

#56
(a) V (narrowing spiral)
(b) II (widening spiral outward)
(c) VI
(d) III
(e) I
(f) IV

Sect 10.4

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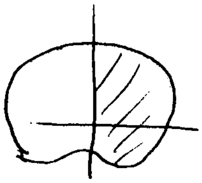
#7 $r = 4 + 3\sin\theta$



$$A = \int_{-\pi/2}^{\pi/2} \frac{1}{2}\left[4+3\sin\theta\right]^2 d\theta$$

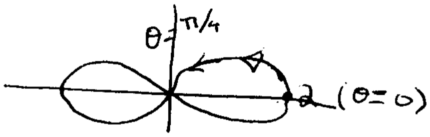
$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$$A = \frac{1}{2}\int_{-\pi/2}^{\pi/2} \left(16 + 24\sin\theta + 9\sin^2\theta\right) d\theta = \frac{1}{2}\int_{-\pi/2}^{\pi/2} 16 + 24\sin\theta + 9\left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$$= \frac{1}{2}\left[16\theta - 24\cos\theta + \frac{9}{2}\theta - \frac{9\sin 2\theta}{4}\right]_{-\pi/2}^{\pi/2} = \frac{1}{2}\left[\left(8\pi + \frac{9\pi}{4}\right) - \left(-8\pi - \frac{9\pi}{4}\right)\right]$$

$$= \frac{1}{2}\left[\frac{41\pi}{2}\right] = \boxed{10.25\pi}$$

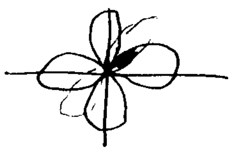
#11 $r^2 = 4\cos 2\theta$



$$A = 4\int_0^{\pi/4} \frac{1}{2} r^2 d\theta = 2\int_0^{\pi/4} 4\cos 2\theta \, d\theta = 8\int_0^{\pi/4} \cos 2\theta \, d\theta$$

$$= 8\left[\frac{\sin 2\theta}{2}\right]\Bigg|_0^{\pi/4} = 8\left[\frac{1}{2} - 0\right] = \boxed{4}$$

#31 $\left.\begin{array}{l} r = \sin 2\theta \\ r = \cos 2\theta \end{array}\right\}$ $\frac{\sin 2\theta}{\cos 2\theta} = 1$ $\tan 2\theta = 1$ $2\theta = \frac{\pi}{4}$ $\theta = \frac{\pi}{8}$



$$A = 8 \cdot 2\int_0^{\pi/8} \frac{1}{2}\sin^2 2\theta \, d\theta$$

$$= 8\int_0^{\pi/8} \frac{1}{2}(1 - \cos 4\theta) d\theta = 4\left(\theta - \frac{\sin 4\theta}{4}\right)\Bigg|_0^{\pi/8} = 4\left(\frac{\pi}{8} - \frac{1}{4} - (0 - 0)\right) = \boxed{\frac{\pi}{2} - 1}$$

Sect 10.5

#15 $9x^2 - 18x + 4y^2 = 27$

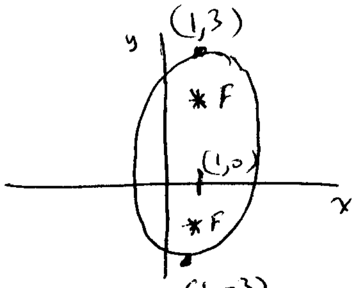
$9(x^2 - 2x) + 4y^2 = 27$

$9(x^2 - 2x + 1) + 4y^2 = 36$ $\Rightarrow$ $\frac{(x-1)^2}{4} + \frac{y^2}{9} = 1$

$a = 3$

$b = 2$ center $(1, 0)$

$c = \sqrt{5}$



vertices $\begin{cases} (1, 3) \\ (1, -3) \end{cases}$

foci $(1, +\sqrt{5})$

$(1, -\sqrt{5})$

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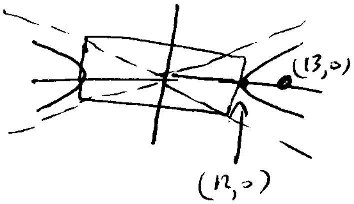
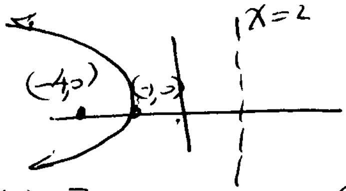
Sect 10.5

#19 Hyp $\dfrac{x^2}{144} - \dfrac{y^2}{25} = 1$ $a = 12$ $b = 5$ $c = \sqrt{144+25} = 13$



center $(0,0)$ vertices $(\pm 12, 0)$ foci (± 13)

asymptotes $y = \pm \frac{5}{12} x$



#33 Parabola focus $(-4,0)$
directrix $x = 2$

$2-(-4) = 6$ $\frac{1}{2}(6) = 3 \Rightarrow p = -3$ (direction negative)

$y^2 = 4p(x+1) = -12(x+1)$ $\boxed{y^2 = -12(x+1)}$

Sect 10.6

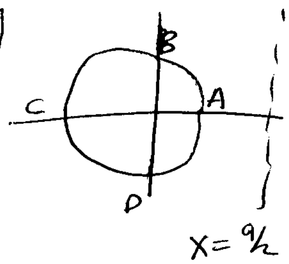
#3 Ellipse eccentricity $= 3/4$
directrix: $x = -5$

$r = \dfrac{ed}{1 - e\cos\theta} = \dfrac{(3/4)5}{1 - \frac{3}{4}\cos\theta}$

$\boxed{r = \dfrac{15}{4 - 3\cos\theta}}$

#13 $r = \dfrac{9}{6 + 2\cos\theta} = \dfrac{9/6}{1 + \frac{2}{6}\cos\theta} = \dfrac{3/2}{1 + \frac{1}{3}\cos\theta} \Rightarrow e = 1/3 < 1$ ellipse

$ed = \frac{3}{2}$ $\frac{1}{3}d = \frac{3}{2}$ $d = \frac{9}{2}$ directrix $\boxed{x = \frac{9}{2}}$



vertices A: $\left(\frac{9}{8}, 0\right) = (r, \theta)$

B: $\left(\frac{3}{2}, \frac{\pi}{2}\right)$

C: $\left(\frac{9}{4}, \pi\right)$

D: $\left(\frac{3}{2}, \frac{3\pi}{2}\right)$

center midway between A & C $\boxed{\left(\frac{9}{16}, \pi\right)}$