Math 132 - Prof. Richard B. Goldstein - Stewart 5e - Chap 12 HW

Sect 12.1

#5 $a_n = \frac{3(-1)^n}{n!}$ $a_1 = \frac{3(-1)}{1!} = -3$, $a_2 = \frac{3(-1)^2}{2!} = \frac{3}{2}$, $a_3 = \frac{3(-1)^3}{3!} = -\frac{1}{2}$, $a_4 = \frac{3(-1)^4}{4!} = \frac{1}{8}$, $a_5 = \frac{3(-1)^5}{5!} = \frac{-1}{40}$

#17 $a_n = \frac{3+5n^2}{n+n^2}$ $\lim_{n\to\infty} \frac{3+5n^2}{n+n^2} = 5$ converges

#35 $a_n = \left(1+\frac{2}{n}\right)^{1/n}$ let $y = \left(1+\frac{2}{n}\right)^{\frac{1}{n}}$

$\ln y = \frac{1}{n}\ln\left(1+\frac{2}{n}\right)$

$\lim_{n\to\infty} \frac{\ln\left(1+\frac{2}{n}\right)}{n} = \frac{\ln(1)}{\infty} = \frac{0}{\infty} = 0$ $e^0 = 1$ $\therefore$ $\underline{\underline{a_n \to 1}}$

#61 $\sqrt{2}$ $a_2 = \sqrt{2a_1}$

$\sqrt{2\sqrt{2}}$ $a_{n+1} = \sqrt{2a_n}$ if $a_n \to x$ then $x = \sqrt{2x}$

$\sqrt{2\sqrt{2\sqrt{2}}}$ $x^2 = 2x \Rightarrow x = 0$ or $\underline{\underline{x = 2}}$

reject

Converges to 2 $a_1 = 1.414$, $a_2 = 1.681, \ldots$, $a_{10} = 1.9986\ldots$, $\to 2$

Sect 12.2

#11 $3 + 2 + \frac{4}{3} + \frac{8}{9} + \ldots$ Geom. $r = \frac{2}{3}$ Since $|r| = \frac{2}{3} < 1 \Rightarrow$ convergent

#23 $\sum_{n=2}^{\infty} \frac{2}{n^2-1} = \sum_{n=2}^{\infty} \frac{2}{(n+1)(n-1)} = \sum_{n=2}^{\infty}\left(\frac{-1}{n+1} + \frac{1}{n-1}\right) = \left(\frac{1}{1} - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{4} - \frac{1}{6}\right) + \left(\frac{1}{5} - \frac{1}{7}\right)$

$\frac{2}{(n+1)(n-1)} = \frac{A}{n+1} + \frac{B}{n-1} = \frac{-1}{n+1} + \frac{1}{n-1}$ left with $\frac{1}{1} + \frac{1}{2} = \underline{\underline{\frac{3}{2}}}$

$2 = A(n-1) + B(n+1) \Rightarrow A = -1$, $B = 1$

#37 $3.\overline{417} = 3.417417\ldots = 3 + \frac{417}{999} = \frac{3414}{999} = \frac{1138}{333}$

Sect 12.3

#3 $\sum_{n=1}^{\infty} \frac{1}{n^4}$ $\int_1^{\infty} \frac{1}{x^4}dx = \int_1^{\infty} x^{-4}dx = \left.\frac{x^{-3}}{-3}\right|_1^{\infty} = \frac{1}{3}$ convergent

#7 $\sum_{n=1}^{\infty} ne^{-n}$ $\int_1^{\infty} xe^{-x}dx$ $u = x$ $dv = e^{-x}dx$ $du = dx$ $v = -e^{-x}$ $\left.-xe^{-x}\right|_1^{\infty} - \int_1^{\infty} -e^{-x}dx$

$e^{-1} + \int_1^{\infty} e^{-x}dx = e^{-1} - \left.(e^{-x})\right|_1^{\infty} = 2e^{-1}$ Conv.

#11 $1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \ldots = \sum_{n=1}^{\infty} \frac{1}{n^3}$ Same as #3 Convergent

Sect 12.3

#21 $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ $\int_2^{\infty} \frac{1}{x \ln x}\,dx$ let $u = \ln x$, $du = \frac{1}{x}dx$, $[2,\infty) \to [\ln 2, \infty)$ ($x \to u$) $\int_{\ln 2}^{\infty} \frac{1}{u}\,du = \ln u \Big|_{\ln 2}^{\infty}$ diverges

Math 132 ②
Prof. R.B. Goldstein
Stewart 5e
Chap 12 HW

Sect 12.4

#3 $\sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$ $0 \le \frac{1}{n^2+n+1} < \frac{1}{n^2}$ and $\sum \frac{1}{n^2}$ is a convergent series $p = 2 > 1$

#5 $\sum_{n=1}^{\infty} \frac{5}{2+3^n}$ $0 \le \frac{5}{2+3^n} < \frac{5}{3^n}$ and $\sum \frac{5}{3^n}$ is geom. series $r = \frac{1}{3} < 1$ Convergent

#9 $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2+1}$ $a_n = \frac{\cos^2 n}{n^2+1}$ $b_n = \frac{1}{n^2}$ $\frac{|a_n|}{|b_n|} = \left| \frac{\cos^2 n}{n^2+1} \cdot n^2 \right| \xrightarrow[n \to \infty]{} 1$

b_n is a convergent p series (note: $|\cos^2 n| \le 1$)

#17 $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$ $a_n = \frac{1}{\sqrt{n^2+1}}$ $b_n = \frac{1}{n}$ (divergent) $\lim_{n \to \infty} \frac{a_n}{b_n} = \lim_{n \to \infty} \frac{n}{\sqrt{n^2+1}} = 1$ $\therefore$ divergent

Sect 12.5

#3 $-\frac{4}{8} + \frac{4}{9} - \frac{4}{10} + \frac{4}{11}$ $a_n = (-1)^{n+1} \frac{4}{7+n}$ Since $\frac{4}{7+n} \to 0$

this series converges

#7 $\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n+1}$ $\frac{3n-1}{2n+1} \longrightarrow \frac{3}{2}$ and $\frac{3}{2}$ is not 0 $\therefore$ divergent

#11 $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+4}$ $\frac{n^2}{n^3+4} \longrightarrow 0$ and alternating $\Rightarrow$ Convergent

#23 $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$ $1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} \mp \ldots$ $\frac{1}{10^2} = .01$ when $n = 10$

Sect 12.6

#5 $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[4]{n}}$ Since $\frac{1}{\sqrt[4]{n}} \to 0$ the series (alternating) converges

however $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n}}$ diverges since $p = \frac{1}{4} \le 1$ p-series divergent absolutely

#11 $\sum_{n=1}^{\infty} \frac{(-1)^n e^{1/n}}{n^3}$ $\lim_{n \to \infty} \left| \frac{e^{1/n}}{n^3} \right| \le \lim_{n \to \infty} \frac{e}{n^3} = 0$ $\therefore$ abs. series is convergent

also since $\left| \frac{(-1)^n e^{1/n}}{n^3} \right| = \left| \frac{e^{1/n}}{n^3} \right| \le \frac{e}{n^3}$ is convergent since $p = 3 > 1$

$\therefore$ absolutely convergent

Math 132 (3)
Prof. R.B. Goldstein
Stewart – 5e
Chap 12 HW

Sect 12.6

#17 $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ Since $\frac{1}{\ln n} \to$ as $n \to \infty$ the Series converges. Absolutely?

Compare to $\sum_{n=2}^{\infty} \frac{1}{n}$ $\quad \frac{1}{n} < \frac{1}{\ln n} \ \forall n$

$\therefore$ does not converge absolutely (conditionally convergent)

Sect 12.7

#9 $\sum_{k=1}^{\infty} k^2 e^{-k}$ $\quad \left|\frac{a_{k+1}}{a_k}\right| = \left|\frac{(k+1)^2 e^{-k+1}}{k^2 e^{-k}}\right| = \left|\frac{(k+1)^2 e^{-1}}{k^2}\right| \to \frac{1}{e} < 1$ as $k \to \infty$

$\therefore$ convergent

#13 $\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$ $\quad \left|\frac{a_{n+1}}{a_n}\right| = \frac{\left|\frac{3^{n+1}(n+1)^2}{(n+1)!}\right|}{\left|\frac{3^n n^2}{n!}\right|} = \left|\frac{3(n+1)^2}{n^2(n+1)}\right| = \left|\frac{3(n+1)}{n^2}\right| \to 0$ as $n \to \infty$

$\therefore$ convergent

#21 $\sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n}$ $\quad \sqrt[n]{a_n} = \sqrt[n]{\left|\frac{(-2)^{2n}}{n^n}\right|} = \sqrt[n]{\frac{2^{2n}}{n^n}} = \frac{4}{n} \to 0$ as $n \to \infty$ $\therefore$ Convergent

Sect 12.8

#5 $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n^3}$ $\quad \frac{\left|\frac{(-1)^n x^{n+1}}{(n+1)^3}\right|}{\left|\frac{(-1)^{n-1} x^n}{n^3}\right|} = \left|\frac{n^3 x}{(n+1)^3}\right| \xrightarrow[n\to\infty]{} |x| < 1$ $\quad R = 1$ Converges for $-1 \le x \le 1$

if $x = 1$ $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$ alt. conv., if $x = -1$ $\sum_{n=1}^{\infty} \frac{(-1)}{n^3}$ also conv $p = 3$

#7 $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ $\quad \frac{\left|\frac{x^{n+1}}{(n+1)!}\right|}{\left|\frac{x^n}{n!}\right|} = \left|\frac{x}{n+1}\right| \to 0 \ \forall n$ $\therefore$ $R = \infty$ $\quad -\infty < x < \infty$

#15 $\sum_{n=0}^{\infty} \sqrt{n}\,(x-1)^n$ $\quad \left|\frac{\sqrt{n+1}\,(x-1)^{n+1}}{\sqrt{n}\,(x-1)^n}\right| \to \left|\sqrt{\frac{n+1}{n}}\,(x-1)\right| \to |x-1| < 1$ $\quad R = 1$

if $x = 2$ $\sum_{n=0}^{\infty} \sqrt{n}$ div., $x = 0$ $\sum_{n=0}^{\infty} \sqrt{n}(-1)^n$ also div. since $\sqrt{n} \not\to 0$

$\therefore$ $0 < x < 2$

#23 $\sum_{n=1}^{\infty} n!\,(2x-1)^n$

$\left|\frac{(n+1)!\,(2x-1)^{n+1}}{n!\,(2x-1)^n}\right| = (n+1)(2x-1) \longrightarrow \infty \quad \forall x$ $\therefore$ $R = 0$

if $x = \frac{1}{2}$ $\sum_{n=0}^{\infty} n!\, 0^n = 0$ Converges only for $x = \frac{1}{2}$

Math 132 (4)
Prof. R.B. Goldstein
Stewart -5e
Chap 12 HW

Sect 12.9

#5 $f(x)=\frac{1}{1-x^3}=1+x^3+x^6+x^9+\dots$ Converges for $|x^3|<1$ or $|x|<1$ or $(-1,1)$

#9 $f(x)=\frac{x}{9+x^2}=\frac{x/9}{1+x^2/9}=\frac{x}{9}-\frac{x^3}{81}+\frac{x^5}{729}-\frac{x^7}{6561}+\dots$

Converges for $\left|\frac{x^2}{9}\right|<1$ or $|x|<3$ $\quad -3<x<3$

#19 $\ln(3+x)=\int\frac{1}{x+3}dx=\frac{1}{3}\int\frac{1}{1+x/3}dx=\frac{1}{3}\int 1-\frac{x}{3}+\frac{x^2}{9}-\frac{x^3}{27}+\dots dx$

$$=\frac{1}{3}\left(x-\frac{x^2}{6}+\frac{x^3}{27}-\frac{x^4}{108}+\frac{x^5}{405}\mp\dots\right)+C$$

Since $\ln(3)=0+C \Rightarrow C=\ln(3)$

$\therefore \ln 3+\frac{x}{3}-\frac{x^2}{18}+\frac{x^3}{81}-\frac{x^4}{324}+\frac{x^5}{1215}-+\dots$ $\quad \left|\frac{x}{3}\right|<1$ or $|x|<3$ $\quad -3<x<3$ $\quad R=3$

#27 $\int_0^{0.2}\frac{1}{1+x^5}dx=\int_0^{0.2}1-x^5+x^{10}-x^{15}\pm\dots dx = x-\frac{x^6}{6}+\frac{x^{11}}{11}-\frac{x^{16}}{16}\pm\Big|_0^{0.2}$

$=0.2-\frac{(0.2)^6}{6}+\frac{(0.2)^{11}}{11}-+\dots$ First two terms: 0.199989 (third or (next) term $\approx 1.9\times10^{-9}$)

Sect 12.10

#7 $f(x)=e^{5x}$

n	0	1	2	3	...
$f^{(n)}(x)$	e^{5x}	$5e^{5x}$	$25e^{5x}$	$125e^{5x}$	...
$f^{(n)}(0)$	5	5	25	125	...

$\sum_{n=0}^{\infty}\frac{5^n}{n!}x^n$ Converges for all x $\quad R=\infty$

#17 $f(x)=\frac{1}{\sqrt{x}}=x^{-1/2}$ $\quad a=9$

n	0	1	2	3
$f^{(n)}(x)$	$x^{-1/2}$	$-\frac{1}{2}x^{-3/2}$	$\frac{3}{4}x^{-5/2}$	$-\frac{15}{8}x^{-7/2}$
$f^{(n)}(9)$	$1/3$	$-\frac{1}{2}\cdot\frac{1}{3^3} = -1/54$	$-\frac{1}{2}\left(-\frac{3}{2}\right)\frac{1}{3^5}$ $1/324$	$\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\frac{1}{3^7}$ $-5/5832$

$$f(x)\approx\frac{1}{3}-\frac{1/54}{1!}(x-9)+\frac{1/324}{2!}(x-9)^2-\frac{5/5832}{3!}(x-9)^3+\dots$$

$$\approx\frac{1}{3}-\frac{1}{54}(x-9)+\frac{1}{648}(x-9)^2-\frac{5}{22,992}(x-9)^3+\dots$$

converges for $\left|\frac{x-9}{9}\right|<1$ $\quad 0<x<18$ $\quad R=9$

#27 $f(x)=x^2e^{-x}=x^2\left(1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}-\dots\right)=x^2-x^3+\frac{x^4}{2!}-\frac{x^5}{3!}+\frac{x^6}{4!}-+\dots$

$R=\infty$

#45 $f(t)=(1+t)^{-1/2}$ similar to #17 $\quad 1-\frac{t}{2}+\frac{3}{8}t^2-\frac{5}{16}t^3+\dots$ let $t=x^3$

$\therefore f(x)=\frac{1}{\sqrt{1+x^3}}=1-\frac{1}{2}x^3+\frac{3}{8}x^6-\frac{5}{16}x^9+\dots$

$\int_0^{0.1}\frac{1}{\sqrt{1+x^3}}dx=\int_0^{0.1}1-\frac{1}{2}x^3+\frac{3}{8}x^6-\frac{5}{16}x^9+\dots dx = x-\frac{1}{8}x^4+\frac{3}{56}x^7-\frac{1}{32}x^{10}+\dots\Big|_0^{0.1}\approx 0.1-\frac{1}{8}(0.1)^4=0.0999875$

next term $\frac{3}{56}(0.1)^7<10^{-8}$