Math 110 – Prof. Richard B. Goldstein – (Chap) Appendix C – Lesson 8th of

Appendix C

C.1 #5 $y = 2x^3$

$y' = 6x^2$

$y' - \frac{3}{x}y = 6x^2 - \frac{3}{x}(2x^3) = 6x^2 - 6x^2 = \boxed{0}$ ✓

#9 $y = 2e^{2x}$

$y' = 2(2e^{2x}) = 4e^{2x}$

$y'' = 8e^{2x}$

$y'' - y' - 2y = 8e^{2x} - 4e^{2x} - 2(2e^{2x}) = 0e^{2x} = \boxed{0}$ ✓

#18 $y = x\ln x^2 + 2x^{3/2} + Cx$

$y = 2x\ln x + 2x^{3/2} + Cx$

$y' = 2\ln x + 2x\left(\frac{1}{x}\right) + 2\left(\frac{3}{2}x^{1/2}\right) + C$

$y' = 2\ln x + 2 + 3x^{1/2} + C$

$y' - \frac{y}{x} = 2\ln x + 2 + 3x^{1/2} + C - \frac{(2x\ln x + 2x^{3/2} + C)}{x}$

$= 2\ln x + 2 + 3x^{1/2} + C - 2\ln x - 2x^{1/2} - C$

$= 2 + x^{1/2} = \boxed{2 + \sqrt{x}}$ ✓

C.2 #1 $\frac{dy}{dx} = \frac{x}{y+3}$ $\quad (y+3)\,dy = x\,dx$ yes separated

#9 $3y^2\frac{dy}{dx} = 1$

$3y^2\,dy = dx$

$\int 3y^2\,dy = \int dx \Rightarrow \boxed{y^3 = x + C}$ or $\boxed{y = \sqrt[3]{x + C}}$

#30 $\frac{dy}{dx} = x^2(1+y) \Rightarrow \frac{dy}{1+y} = x^2\,dx \Rightarrow \int\frac{dy}{1+y} = \int x^2\,dx$

$y(0) = 3$

$\ln(1+y) = \frac{x^3}{3} + C$ $\quad \ln 4 = 0 + C$ $\quad C = \ln 4$

$\ln(1+y) = \frac{x^3}{3} + \ln 4$ $\quad e^{\ln(1+y)} = e^{\frac{x^3}{3} + \ln 4}$

$1 + y = 4e^{x^3/3} \Rightarrow \boxed{y = 4e^{\frac{x^3}{3}} - 1}$

#37 $T = T_0 + Ce^{kt}$ $\quad T_0 = 90$

When $t = 0$, $T = 1500$ $\quad 1500 = 90 + C \Rightarrow C = 1410$

When $t = 1$, $T = 1120$ $\quad 1120 = 1410e^k + 90$ $\quad e^k = \frac{1030}{1410}$ $\quad k = \ln\frac{1030}{1410} = -0.31403$

$\boxed{T = 90 + 1410e^{-0.31403t}}$

$T(5) = 90 + 1410(0.20801\ldots) = \boxed{383.3°F}$

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C.3

#7 $\frac{dy}{dx} + 3y = 6$ $\quad e^{\int 3\,dx} = e^{3x}$

$e^{3x}\frac{dy}{dx} + 3e^{3x}y = 6e^{3x}$

$\frac{d}{dx}(e^{3x}y) = 6e^{3x} \Rightarrow e^{3x}y = \int 6e^{3x}\,dx = 2e^{3x} + c$

$\boxed{y = 2 + ce^{-3x}}$

#11 $\frac{dy}{dx} + \frac{y}{x} = 3x + 4$ $\quad e^{\int \frac{1}{x}\,dx} = e^{\ln x} = x$

$x\frac{dy}{dx} + y = 3x^2 + 4x$ $\quad \frac{d}{dx}(xy) = 3x^2 + 4x$

$xy = \int 3x^2 + 4x\,dx = x^3 + 2x^2 + c$

$\boxed{y = x^2 + 2x + \frac{c}{x}}$

#31 $y' + 3x^2y = 3x^2$ $\quad e^{\int 3x^2\,dx} = e^{x^3}$

$\frac{d}{dx}(e^{x^3}y) = 3x^2e^{x^3} \Rightarrow e^{x^3}y = \int 3x^2e^{x^3}\,dx = e^{x^3} + c$

$\boxed{y = 1 + ce^{-x^3}}$ $\quad \boxed{y = 1 + 5e^{-x^3}}$

$y = 6 \quad x = 0$ $\quad y = 1 + ce^0 = 1 + c = 6, \; c = 5$

C.4

#7 $\frac{dA}{dt} = kA \Rightarrow A(t) = A_0e^{kt}$

$A(5) = 2983.65$ $\quad 2983.65 = 2000e^{5k}$ $\quad e^{5k} = 1.491825$

$A(0) = 2000$ $\quad 5k = 0.4$ $\quad k = 0.08$ (8%)

$A(10) = 2000e^{.08(10)} = 2000e^{0.8} = \boxed{4451.08}$

#15 $\frac{dN}{dt} = kN(L - N)$ $\quad L = 500$

$N(0) = 100$

$N(4) = 200$

$\int \frac{dN}{N(500 - N)} = \int k\,dt$

$\frac{1}{500}\int \left(\frac{1}{N} + \frac{1}{500 - N}\right)dN = kt + C$

$\ln N - \ln(500 - N) = 500kt + 500C$

$\frac{N}{500 - N} = Ae^{500kt} \quad (e^{500C} = A)$

$N = \frac{500Ae^{500kt}}{1 + Ae^{500kt}} = \frac{500A}{A + e^{-500kt}}$

$N(0) = \frac{500A}{A + 1} = 100$ $\quad 500A = 100A + 100$ $\quad \boxed{A = 0.25}$

$400A = 100$

$N(t) = \frac{125}{0.25 + e^{-500kt}}$ $\quad N(4) = 200 = \frac{125}{0.25 + e^{-2000k}} \Rightarrow k = \frac{\ln(8/3)}{2000}$

$= 4.904 \times 10^{-4}$

$50 + 200e^{-2000k} = 125 \Rightarrow e^{-2000k} = 0.375$

$N(t) = \frac{125}{0.25 + e^{-0.2452t}} = \boxed{\frac{500}{1 + 4e^{-0.2452t}}}$