Math 110 – Prof. Richard B. Goldstein – Larson 8th Ed – Chap 6 HW

Sect 7.1

#15 $(-1,-5,7),\ (-3,4,-4)$ $\quad d = \sqrt{[-1-(-3)]^2 + (-5-4)^2 + (7-(-4))^2} = \sqrt{4+81+121} = \boxed{\sqrt{206}}$

#33 Opposite pts. on sphere $(2,1,3),\ (1,3,-1)$

$D = \sqrt{(2-1)^2 + (1-3)^2 + [3-(-1)]^2} = \sqrt{1+4+16} = \sqrt{21} = 2r \qquad r = \frac{\sqrt{21}}{2}$

Center $\left(\frac{2+1}{2}, \frac{1+3}{2}, \frac{3-1}{2}\right) = (1.5, 2, 1)$

$$\boxed{(x-1.5)^2 + (y-2)^2 + (z-1)^2 = \frac{21}{4}}$$

#43 $x^2+y^2+z^2 - 2x + 6y + 8z + 1 = 0$

$(x-1)^2 + (y+3)^2 + (z+4)^2 = -1+1+9+16 = 25$

$\boxed{\text{center } (1,-3,-4) \quad \text{radius } r = \sqrt{25} = 5}$

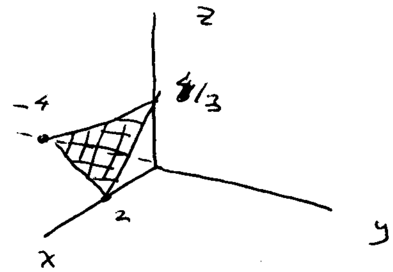
Sect 7.2

#5 $2x - y + 3z = 4$

$2x = 4,\ x = 2$

$-y = 4,\ y = -4$

$3z = 4,\ z = 4/3$



#15 $(1,5,-4),\ 3x - y + 2z = 6$

$$D = \frac{|3(1) + (-1)(5) + (2)(-4) - 6|}{\sqrt{3^2+1^2+2^2}} = \boxed{\frac{16}{\sqrt{14}} = \frac{8}{7}\sqrt{14}}$$

#23 $x - 5y - z = 1$

$5x - 25y - 5z = -3$

$\frac{1}{5} = \frac{-5}{-25} = \frac{-1}{-5} \neq \frac{1}{-3}$ $\quad \therefore$ parallel (but not coincidental)

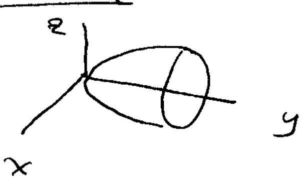
#26 $x + 3y + z = 7$

$x - 5z = 0$

not parallel $\quad (1)(1) + (3)(0) + (1)(-5) = -4 \neq 0$

$\therefore$ not perpendicular

#35 $4x^2 - 4y + z^2 = 0$ or $4y = 4x^2 + z^2$ $\quad \boxed{\text{Paraboloid (d)}}$



#49 $2x^2 - y^2 + 2z^2 = -4$ $\quad \boxed{\text{hyperboloid of two sheets}}$

or $-2x^2 + y^2 - 2z^2 = 4$

two negatives

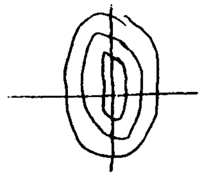
Sect 7.3

#5 $h(x,y,z) = \dfrac{xy}{z}$

(a) $h(2,3,9) = \dfrac{2(3)}{9} = \dfrac{6}{9} = \boxed{\dfrac{2}{3}}$

(b) $h(1,0,1) = \dfrac{1(0)}{1} = \boxed{0}$

#29 $f(x,y) = x^2 + \dfrac{y^2}{4} = k$ are ellipses



Sect 7.4

#9 $z = x^2 e^{2y}$ $\qquad \dfrac{\partial z}{\partial x} = \boxed{2xe^{2y}} \qquad \dfrac{\partial z}{\partial y} = x^2 e^{2y} \cdot 2 = \boxed{2x^2 e^{2y}}$

#17 $g(x,y) = 3xy^2 e^{y-x} \qquad g_x(x,y) = 3y^2 e^{y-x} + 3xy^2 e^{y-x}(-1) = \boxed{(3y^2 - 3xy^2)\, e^{y-x}}$

#37 $w = 2xz^2 + 3xyz - 6y^2 z$

$w_x = 2z^2 + 3yz \Big|_{(1,-1,2)} = 2(2)^2 + 3(-1)(2) = \boxed{2}$

$w_y = 3xz - 12yz \Big|_{(1,-1,2)} = 3(1)(2) - 12(-1)(2) = \boxed{30}$

$w_z = 4xz + 3xy - 6y^2 \Big|_{(1,-1,2)} = 4(1)(2) + 3(1)(-1) - 6(-1)^2 = \boxed{-1}$

#55 $f(x,y) = x^4 - 3x^2y^2 + y^2 \quad (1,0)$

$f_x = 4x^3 - 6xy^2$ $\qquad f_{xy} = -12xy \Big|_{(1,0)} = \boxed{0}$

$f_{xx} = 12x^2 - 6y^2 \Big|_{(1,0)} = \boxed{12}$

same

$f_y = -6x^2y + 2y$

$f_{yx} = -12xy \Big|_{(1,0)} = \boxed{0}$ $\qquad f_{yy} = -6x^2 + 2 \Big|_{(1,0)} = \boxed{-4}$

#61 $f(x,y) = 200x^{0.7}y^{0.3} \quad (1000, 500)$

$\dfrac{\partial f}{\partial x} = 140x^{-0.3}y^{0.3} \Big|_{(1000,500)} = \boxed{113.71}$ $\qquad \dfrac{\partial f}{\partial y} = 60x^{0.7}y^{-0.7} \Big|_{(1000,500)} = \boxed{97.47}$

Sect 7.5

#1 $f(x,y) = x^2 - y^2 + 4x - 8y - 11$

$\left.\begin{array}{l} f_x = 2x + 4 = 0 \\ f_y = -2y - 8 = 0 \end{array}\right\} \Rightarrow \begin{array}{l} x = -2 \\ y = -4 \end{array}$

$f_{xx} = 2$, $f_{xy} = 0$, $f_{yy} = -2$

$d = (2)(-2) - 0^2 = -4$ $\qquad \boxed{\text{Saddle pt}} \qquad \boxed{f(-2,-4) = 1}$

#9 $f(x,y) = -5x^2 + 4xy - y^2 + 16x + 10$

$\left.\begin{array}{l} f_x = -10x + 4y + 16 = 0 \\ f_y = 4x - 2y = 0 \end{array}\right\} \Rightarrow \boxed{x = 8,\ y = 16}$

$f_{xx} = -10$, $f_{xy} = 4$, $f_{yy} = -2$

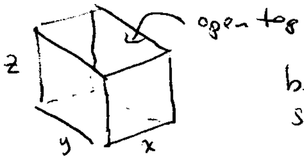
$d = (-10)(-2) - (4)^2 = +4$

$f_{xx} < 0$ $\qquad \boxed{\text{relative maximum}}$

$\boxed{f(8,16) = 74}$

Sect 7.5

#23 $f_{xx}(x_0,y_0) = -9$, $f_{yy}(x_0,y_0) = 6$, $f_{xy}(x_0,y_0) = 10$ $\Rightarrow$ $d = (-9)(+6) - (10)^2 = -154 < 0$ $\boxed{\text{Saddle point}}$

#45 

$V = xyz = 6 \Rightarrow z = \frac{6}{xy}$

bottom \$0.15/sq ft
sides 0.10/sq ft

$C = [2xz + 2yz]0.10 + [xy]0.15$
$= (2x+2y)\left(\frac{6}{xy}\right)0.1 + 0.15xy$

$C_1 = \frac{1.2}{y} + \frac{1.2}{x} + 0.15xy$

$C_x = -\frac{1.2}{x^2} + 0.15y = 0$ $\quad y = \frac{8}{x^2}$
$C_y = -\frac{1.2}{y^2} + 0.15x = 0$ $\quad x = \frac{8}{y^2}$
$y = \frac{8}{\left(\frac{8}{y^2}\right)^2} = \frac{y^4}{8}$ $\quad y = 0, 2$

$\boxed{x = y = 2}$ $\quad C_1 = \frac{1.2}{2} + \frac{1.2}{2} + 0.15(2)(2) = 0.6 + 0.6 + 0.6 = \boxed{\$1.80}$

Sect 7.6

#5 Max $x^2 - y^2$ s.t. $2y - x^2 = 0$

$F = x^2 - y^2 + \lambda(2y - x^2)$
$F_x = 2x - 2x\lambda = 0 = 2x(1-\lambda) = 0$
$F_y = -2y + 2\lambda = 0 \quad y = \lambda$
$2y = x^2$

$x = 0, y = 0, \lambda = 0$
$x = \sqrt{2}, y = 1, \lambda = 1$

$f(0,0) = 0^2 - 0^2 = 0$
$f(\sqrt{2},0) = 2 - 0 = 2$
$f(-\sqrt{2},0) = 2 - 0 = 2$ } max $\quad (\sqrt{2},0) \quad (-\sqrt{2},0)$

#13 Min $2x^2 + 3y^2 + 2z^2$ s.t. $x + y + z - 24 = 0$

$F = 2x^2 + 3y^2 + 2z^2 + \lambda(x + y + z - 24)$
$F_x = 4x + \lambda = 0 \quad x = -\frac{\lambda}{4}$
$F_y = 6y + \lambda = 0 \quad y = -\frac{\lambda}{6}$
$F_z = 4z + \lambda = 0 \quad z = -\frac{\lambda}{4}$

$-\frac{\lambda}{4} - \frac{\lambda}{6} - \frac{\lambda}{4} = 24$
$\lambda = -36$

$x = 9, y = 6, z = 9$ has a $\boxed{\text{minimum}}$ at $(9,6,9)$ of $\boxed{f(9,6,9) = 432}$

***#21** Max xyz s.t. $x^2 + z^2 = 5$, $x - 2y = 0$

$F = xyz + \lambda(x^2 + z^2 - 5) + \mu(x - 2y)$
$F_x = yz + 2x\lambda + \mu = 0$
$F_y = xz - 2\mu = 0$
$F_z = xy + 2z\lambda = 0$
$x^2 + z^2 = 5 \quad \boxed{x = 2y}$

$\mu = \frac{xz}{2}$
$\lambda = -\frac{xy}{2z}$

after much work

$f\left(\sqrt{\frac{10}{3}}, \frac{1}{2}\sqrt{\frac{10}{3}}, \sqrt{\frac{5}{3}}\right) = \frac{5\sqrt{15}}{9}$

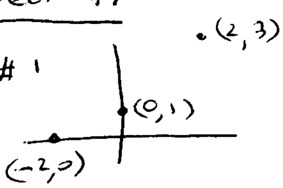
***#33** Max xyz s.t. $x + 2y + 2z = 108$

$F = xyz + \lambda(x + 2y + 2z - 108)$
$F_x = yz + \lambda = 0$
$F_y = xz + 2\lambda = 0$
$F_z = xy + 2\lambda = 0$

$x = 2y$
$y = z$
$x = 36, y = 18, z = 18$

$\boxed{V \text{ max for } 36'' \times 18'' \times 18''}$

Sect 7.7

#1 

$\Sigma x_i = 0$
$\Sigma y_i = 4$
$\Sigma x_i y_i = 6$
$\Sigma x_i^2 = 8$
$n = 3$

$a = \frac{3(6) - 0(4)}{3(8) - 0^2} = \frac{3}{4}$

$b = \frac{1}{3}\left[4 - \frac{3}{4}(0)\right] = \frac{4}{3}$

$\boxed{\hat{y} = \frac{3}{4}x + \frac{4}{3}}$

$\hat{y}(-2) = -\frac{1}{6}$ etc

Sum $S_q = \left(-\frac{1}{6} - 0\right)^2 + \left(\frac{4}{3} - 1\right)^2 + \left(\frac{17}{6} - 3\right)^2$
$= \boxed{\frac{1}{6}}$

Math 110 (4)
Prof. R. B Goldstein
Larson 8th Ed–Chap 6 HW

Sect 7.7

#27

Price x	1.00	1.25	1.50
Demand y	450	375	330

$n=3$, $\Sigma x = 3.75$, $\Sigma y = 1155$, $\Sigma x^2 = 4.8125$, $\Sigma xy = 1413.75$ $\Big\}$ $b = 685$, $a = -240$

$\boxed{\hat{y} = 685 - 240x}$

$\boxed{\hat{y}(1.4) = 349}$

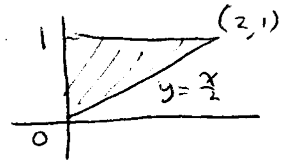
$500 = 685 - 240x \Rightarrow x = \frac{185}{240} \approx \boxed{0.77}$

Sect 7.8

#15 $\displaystyle\int_0^1\int_0^y (x+y)\,dx\,dy = \int_0^1 \frac{x^2}{2} + xy \; dy = \int_0^1 \frac{3y^2}{2}\,dy = \frac{y^3}{2}\Big|_0^1 = \boxed{\frac{1}{2}}$

#19 $\displaystyle\int_0^2\int_0^{\sqrt{1-y^2}} -5xy\,dx\,dy = \int_0^2 -\frac{5x^2y}{2}\Big|_0^{\sqrt{1-y^2}} dy = -\frac{5}{2}\int_0^2 (y-y^3)\,dy = -\frac{5}{2}\left(\frac{y^2}{2}-\frac{y^4}{4}\right)\Big|_0^2 = -\frac{5}{2}(2-4) = \boxed{5}$

#29 $\displaystyle\int_0^2\int_{x/2}^1 dy\,dx = \int_0^2 \left(1-\frac{x}{2}\right)dx = x - \frac{x^2}{4}\Big|_0^2 = 1$



also $\displaystyle\int_0^1\int_0^{2y} dx\,dy$

Sect 7.9

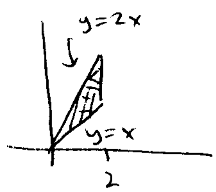
#3 $\displaystyle\int_0^1\int_y^{\sqrt{y}} x^2y^2\,dx\,dy$

$x=\sqrt{y}$, $x=y$ $\|$ $y=x^2$, $y=x$

$\displaystyle\int_0^1 \frac{x^3y^2}{3}\Big|_{x=y}^{x=\sqrt{y}} dy = \int_0^1 \frac{y^{7/2}}{3} - \frac{y^5}{3}\,dy$

$= \frac{2y^{9/2}}{27} - \frac{y^6}{18} = \frac{2}{27} - \frac{1}{18} = \boxed{\frac{1}{54}}$

#11



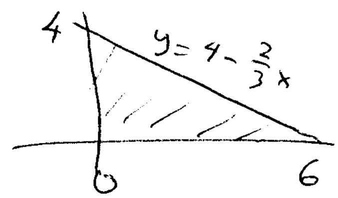
$\displaystyle\int_0^2\int_x^{2x} \frac{y}{x^2+y^2}\,dy\,dx = \int_0^2 \frac{\ln(x^2+y^2)}{2}\Big|_{y=x}^{y=2x} dx = \int_0^2 \frac{\ln(5x^2)}{2} - \frac{\ln(2x^2)}{2}\,dx$

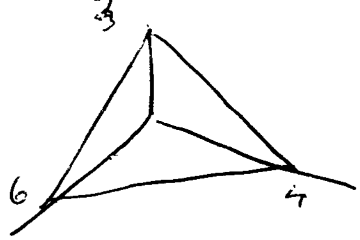
$\displaystyle = \int_0^2 \frac{1}{2}\ln\left(\frac{5x^2}{2x^2}\right)dx = \int_0^2 \frac{1}{2}\ln\left(\frac{5}{2}\right)dx = \frac{1}{2}\ln\left(\frac{5}{2}\right)x\Big|_0^2 = \boxed{\ln\left(\frac{5}{2}\right)}$

#17 $2x+3y+4z=12$

$\displaystyle V = \int_0^6\int_0^{4-\frac{2}{3}x} \left(\frac{12-2x-3y}{4}\right)dy\,dx = \int_0^6 \frac{1}{4}\left(12y - 2xy - \frac{3}{2}y^2\right)\Big|_0^{4-\frac{2}{3}x} dx$

$\displaystyle = \int_0^6 \left(\frac{1}{6}x^2 - 2x + 6\right)dx = \frac{x^3}{18} - x^2 + 6x\Big|_0^6 = \boxed{12}$





$V = \frac{1}{6}(6)(3)(4) = \boxed{12}$

$\frac{1}{6}(a)(b)(c)$