Math 110 - Prof. Richard B. Goldstein - Larson 8th Ed - Chap 6 HW

Sect 6.1

#3 $\int \underbrace{x \overbrace{\ln 2x}^{u} dx}_{dv}$ let $u = \ln 2x = \ln 2 + \ln x$, $dv = x\,dx$, then $du = \frac{1}{x}dx$ and $v = \frac{x^2}{2}$ $\Rightarrow$ $\frac{x^2}{2}\ln(2x) - \int \frac{x^2}{2}\frac{1}{x}dx = \boxed{\frac{x^2}{2}\ln(2x) - \frac{x^2}{4} + c}$

#7 $\int \underbrace{x^2}_{u} \underbrace{e^{-x}dx}_{dv}$ let $u = x^2$ then $du = 2x\,dx$, $dv = e^{-x}dx$ then $v = -e^{-x}$ $\Longrightarrow$ $-x^2e^{-x} - \int(-e^{-x})2x\,dx$

$= -x^2e^{-x} + 2\int xe^{-x}dx$ let $u = x$ then $du = dx$, $dv = e^{-x}dx$ then $v = -e^{-x}$ $\quad -x^2e^{-x} - 2xe^{-x} - 2\int e^{-x}dx$

$= -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + c = \boxed{-(x^2+2x+2)e^{-x} + c}$

#29 $\int \frac{(\ln x)^2}{x}dx$ $u = \ln x$, $du = \frac{1}{x}dx$ $\Longrightarrow$ $\int u^2\,du = \frac{u^3}{3} + c = \boxed{\frac{(\ln x)^3}{3} + c}$ substitution not integ by parts

#43 $\int_1^e x^5 \ln x\,dx$ $u = \ln x$, $du = \frac{1}{x}dx$, $dv = x^5dx$, $v = \frac{x^6}{6}$ $\Longrightarrow$ $\frac{x^6 \ln x}{6}\Big|_1^e - \int_1^e \underbrace{\frac{x^6}{6}\frac{1}{x}}_{x^5/6}dx$

$= \frac{e^6}{6} - \left(\frac{x^6}{36}\right)\Big|_1^e = \frac{e^6}{6} - \frac{e^6}{36} + \frac{1}{36} = \boxed{\frac{1}{36} + \frac{5e^6}{36}} \approx 56.06$

#67 $\bar{M}_{(1,2)} = \dfrac{\int_1^2 (1 + 1.6t\ln t)\,dt}{2-1}$ $u = \ln t$, $dv = t\,dt$, $du = \frac{1}{t}dt$, $v = \frac{t^2}{2}$ $\quad t\Big|_1^2 + 1.6\left(\frac{t^2}{2}\ln t\right)\Big|_1^2 - 1.6\int_1^2 \frac{t}{2}dt$

$= 1 + 1.6(2\ln 2) - 1.6\frac{t^2}{4}\Big|_1^2$

$= 1 + 3.2\ln 2 - 1.6 + 0.4$

$\boxed{-0.2 + 3.2\ln 2} \approx \boxed{2.018}$

$\bar{M}_{(3,4)} = \dfrac{\left[t + 1.6\left(\frac{t^2}{2}\ln t\right) - 1.6\frac{t^2}{4}\right]_3^4}{4-3} = \boxed{12.8\ln 4 - 1.8 - 7.2\ln 3} \approx \boxed{8.035}$

#75 $C = 150{,}000 + 75{,}000t$
$r = 4\%$
4 yrs

(A) $\int_0^4 (150000 + 75000t)\,dt = 150000t + 37500t^2\Big|_0^4 = \boxed{\$1{,}200{,}000}$

(B) Present Value $= \int_0^4 (150000 + 75000t)e^{-0.04t}\,dt$ = (integrate by parts) $-1{,}875{,}000\,e^{-0.04t}(t + 27)\Big|_0^4$

$= 50{,}625{,}000 - 58{,}125{,}000\,e^{-0.16} = \boxed{\$1{,}094{,}142.27}$

Sect 6.2

#5 $\dfrac{4x-13}{x^2-3x-10} = \dfrac{4x-13}{(x-5)(x+2)} = \dfrac{A}{x-5} + \dfrac{B}{x+2}$

$4x-13 = A(x+2) + B(x-5)$ let $x=-2$ $-21=-7B$ $B=3$
let $x=5$ $7=7A$ $A=1$

$$\boxed{\frac{1}{x-5} + \frac{3}{x+2}}$$

#23 $\displaystyle\int \frac{5-x}{2x^2+x-1}\,dx$ $\dfrac{5-x}{2x^2+x-1} = \dfrac{A}{2x-1} + \dfrac{B}{x+1}$ let $x=-1$ $6=-3B$ $B=-2$
$x=\frac{1}{2}$ $\frac{9}{2}=\frac{3A}{2}$ $A=3$

$5-x = A(x+1) + B(2x-1)$

$$\int \frac{3}{2x-1} - \frac{2}{x+1}\,dx = \boxed{\frac{3}{2}\ln(2x-1) - 2\ln(x+1) + C}$$

#35 $\displaystyle\int_1^5 \frac{x-1}{x^2(x+1)}\,dx$ $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+1} = \dfrac{x-1}{x^2(x+1)}$ at $x=0$ $B=-1$
$x=-1$ $C=-2$
$x=1$ $2A+2B+C=0$
$Ax(x+1) + B(x+1) + Cx^2 = x-1$ $2A-4=0$, $A=2$

$$\int_1^5 \frac{2}{x} - \frac{1}{x^2} - \frac{2}{x+1}\,dx = 2\ln x + \frac{1}{x} - 2\ln(x+1)\Big|_1^5$$

$$=\left(2\ln 5 + \frac{1}{5} - 2\ln 6\right) - (1 - 2\ln 2) \cong \boxed{0.2217}$$

#54 $t = 5010\displaystyle\int \frac{dx}{(x+1)(500-x)}$ $\dfrac{5010}{(x+1)(500-x)} = \dfrac{A}{x+1} + \dfrac{B}{500-x}$ let $x=500$
$5010 = 501B$ $\underline{B=10}$
let $x=-1$
$5010 = A(500-x) + B(x+1)$ $5010 = 501A$ $\underline{A=10}$

$$t = \int \frac{10}{x+1} + \frac{10}{500-x}\,dx = 10\ln(x+1) - 10\ln(500-x) + C$$

$$0 = 10\ln 2 - 10\ln 499 + C \Rightarrow C = 61.433$$

(A) $x=0.75$ $10\ln 1.75 - 10\ln 499.25 + 61.433 = \boxed{4.898}$ hrs

(B) $100 = 10\ln\left(\dfrac{x+1}{500-x}\right) + 61.433$ $\ln\left(\dfrac{x+1}{500-x}\right) = 3.8567$

$\dfrac{x+1}{500-x} = e^{3.8567} = 47.309$ $x+1 = 23654.5 - 47.309x$
$48.309x = 23653.5$ $\Rightarrow \boxed{x = 489.6}$

Math 110 (3)

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Sect 6.3

\#5 $\displaystyle\int \frac{2x}{\sqrt{x^4-9}}$

FORMULA #25 $\displaystyle\int \frac{1}{\sqrt{u^2 \pm a^2}}\,du = \ln\left|u + \sqrt{u^2 \pm a^2}\right| + c$

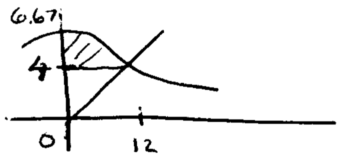
let $u = x^2$, $du = 2x\,dx$

$\displaystyle\int \frac{du}{\sqrt{u^2-9}} = \boxed{\ln\left|x^2 + \sqrt{x^2-9}\right| + c}$

\#29 $\displaystyle\int \frac{1}{x^2\sqrt{1-x^2}}\,dx$ FORMULA #33 $\boxed{-\dfrac{\sqrt{1-x^2}}{x} + c}$

\#60 Demand $p = \dfrac{60}{\sqrt{x^2+81}}$

supply $p = \dfrac{x}{3}$



$\displaystyle C.S. = \int_0^{12} \frac{60}{\sqrt{x^2+81}} - 4 \quad dx$

$\displaystyle P.S. = \int_0^{12} \left(4 - \frac{x}{3}\right) dx$

$\displaystyle C.S. = 60 \ln\left(x + \sqrt{x^2+81}\right) - 4x \Big|_0^{12} = 60 \ln 3 - 48 = \boxed{17.917}$

$\displaystyle P.S. = 4x - \frac{x^2}{6}\Big|_0^{12} = 48 - 24 = \boxed{24}$

Sect 6.4

\#9 $\displaystyle\int_0^4 \sqrt{x}\,dx$ $(n = 8)$ actual $\displaystyle \frac{x^{3/2}}{3/2}\Big|_0^4 = \frac{16}{3}$

x	0	0.5	1	1.5	2	2.5	3	3.5	4
$f(x)$	0	0.7071	1	1.2247	1.4142	1.5811	1.7321	1.8708	2

$T_8 = \frac{0.5}{2}\left[f(0) + 2\{f(0.5) + f(1) + f(1.5) + f(2) + f(2.5) + f(3) + f(3.5) + f(4)\right]$

$= \frac{1}{4}(21.0602) = \boxed{5.2651}$

$S_8 = \frac{0.5}{3}\left[f(0) + 4f(0.5) + 2f(1) + 4f(1.5) + 2f(2) + 4f(2.5) + 2f(3) + 4f(3.5) + f(4)\right]$

$= \frac{1}{6}\left[31.8278\right] = \boxed{5.3046}$

\#19 $\displaystyle\int_0^1 \sqrt{1-x^2}\,dx$ $n = 4$ $h = \frac{1-0}{4} = 0.25$

x	0	0.25	0.5	0.75	1
$f(x)$	1	0.9682	0.8660	0.6614	0

$T_4 = \frac{0.25}{2}\left[1 + 2(0.9682 + 0.8660 + 0.6614) + 0\right] = \frac{1}{8}(5.9912) = \boxed{0.7489}$

$S_4 = \frac{0.25}{2}\left[1 + 4(0.9682) + 2(0.8660) + 4(0.6614) + 0\right] = \frac{9.2504}{4} = \boxed{0.7709}$

actual quarter of a unit circle $\frac{\pi}{4} = 0.7854$

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Sect 6.5

#15 $\displaystyle\int_5^\infty \frac{x}{\sqrt{x^2-16}}\,dx$

$u = x^2-16$
$u(5) = 9$
$u(\infty) = \infty$
$du = 2x\,dx$
$\frac{du}{2} = x\,dx$

$\displaystyle\int_9^\infty \frac{\frac{du}{2}}{\sqrt{u}} = \sqrt{u}\Big|_9^\infty$

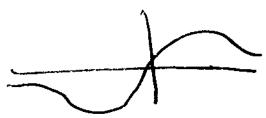
diverges at upper limit

#21 $\displaystyle\int_{-\infty}^\infty 2x e^{-3x^2}\,dx$

$u = 3x^2$
$du = 6x\,dx$
$\frac{1}{3}du = 2x\,dx$

asymmetric $f(-x) = -f(x)$

$\therefore \boxed{0}$



note $\displaystyle\int_0^\infty 2x e^{-3x^2}\,dx = -\frac{1}{3}e^{-u}\Big|_0^\infty = 0 + \frac{1}{3} = \boxed{\frac{1}{3}}$

must converge

$\frac{1}{3} - \frac{1}{3} = 0$

#25 $\displaystyle\int_0^9 \frac{1}{\sqrt{9-x}}\,dx$

$u = 9-x$
$-du = dx$
$u(0) = 9$
$u(9) = 0$

$\displaystyle\int_9^0 u^{-1/2}(-du) = \int_0^9 u^{1/2}\,du$

$= 2\sqrt{u}\Big|_0^9 = 2(3) - 2(0) = \boxed{6}$

limit is used at $u \to 0$