Math 109/110 – Prof. Richard B. Goldstein – Larson 8th Ed. Chap 5 HW

Sect 5.1

#3 $\int\left(4x^3-\frac{1}{x^2}\right)dx = x^4+\frac{1}{x}+c$ $\quad \frac{d}{dx}\left(x^4+x^{-1}+c\right)=4x^3-x^{-2}+0=4x^3-\frac{1}{x^2}$ ✓

#23 $\int \frac{1}{x\sqrt{x}}dx=\int x^{-3/2}dx = \frac{x^{-1/2}}{-1/2}+c = \boxed{-\frac{2}{\sqrt{x}}+c}$

#34 $\int\left(\sqrt[4]{x^3}+1\right)dx=\int x^{3/4}+1\,dx = \frac{x^{7/4}}{7/4}+x+c = \boxed{\frac{4}{7}x^{7/4}+x+c}$

#53 $f'(x)=\frac{2-x}{x^3}=2x^{-3}-x^{-2}$ $\quad f(2)=\frac{3}{4}$ $\quad f(x)=\int 2x^{-3}-x^{-2}dx = \frac{2x^{-2}}{-2}-\frac{x^{-1}}{-1}+c$

$f(x)=-\frac{1}{x^2}+\frac{1}{x}+c$ $\quad f(2)=-\frac{1}{4}+\frac{1}{2}+c=\frac{3}{4} \Rightarrow c=1/2$ $\quad \therefore \boxed{f(x)=-\frac{1}{x^2}+\frac{1}{x}+\frac{1}{2}}$

#61 $f''(x)=x^{-2/3} \Rightarrow f'(x)=\frac{x^{1/3}}{1/3}+c = 3x^{1/3}+c$ $\quad f'(8)=6 \Rightarrow 3\cdot 2+c=6 \therefore c=0$

$\Rightarrow f(x)=\frac{3x^{4/3}}{4/3}+D = \frac{9}{4}x^{4/3}+D$ $\quad f(0)=0=0+D \therefore D=0$

$\boxed{f(x)=\frac{9}{4}x^{4/3}}$

#75 Marginal Cost $\frac{dC}{dx}=2x-12 \Rightarrow C(x)=x^2-12x+K$ $\quad C(0)=125$

(a) $\therefore K=125$ ← fixed cost (rent, machinery)

$\boxed{C'(x)=x^2-12x+125}$

avg cost $\bar{C}(x)=\frac{C(x)}{x}=\frac{x^2-12x+125}{x}=\boxed{x-12+\frac{125}{x}}$

(b) $C(50)=50^2-12(50)+125=2500-600+125=\boxed{\$2,025}$

(c) $2025-125=\boxed{\$1,900}$ is the variable cost (ex. material, labor)

Sect 5.2

#3 $\int\sqrt{1-x^2}\,(-2x)\,dx$ $\quad$ let $u=1-x^2$ $\quad \frac{du}{dx}=-2x$

#11 $\int\sqrt{4x^2-5}\,(8x)\,dx$ $\quad$ let $u=4x^2-5$ $\quad \frac{du}{dx}=8x \Rightarrow \int u^{1/2}du=\frac{u^{3/2}}{3/2}+c$

$f(x)=\frac{2}{3}u^{3/2}=\boxed{\frac{2}{3}(4x^2-5)^{3/2}+c}$ $\quad f'(x)=\frac{2}{3}\cdot\frac{3}{2}(4x^2-5)^{1/2}\cdot 8x=\sqrt{4x^2-5}\,(8x)$ ✓

Sect 5.2

#19 $\int \frac{x+1}{(x^2+2x-3)^2}\,dx$ let $u = x^2+2x-3$ $\frac{du}{dx} = 2x+2$ $\int \frac{\frac{1}{2}\frac{du}{dx}\,dx}{u^2} = \int \frac{1}{2}u^{-2}\,du$

$= +\frac{u^{-1}}{-2} = \frac{-1}{2u} = \boxed{-\frac{1}{2(x^2+2x-3)} + c}$

#47 $f(0) = \frac{4}{3}$ $f'(x) = x\sqrt{1-x^2}$ $\int (1-x^2)^{1/2} x\,dx$ $u = 1-x^2$ $\frac{du}{dx} = -2x$

$\int u^{1/2}\left(-\frac{1}{2}du\right) = \frac{-\frac{1}{2}u^{3/2}}{3/2} + c = -\frac{u^{3/2}}{3} + c = \boxed{-\frac{(1-x^2)^{3/2}}{3} + c}$

$f(0) = -\frac{1}{3} + c = \frac{4}{3}$ $c = \frac{5}{3}$ $\therefore \boxed{f(x) = \frac{5}{3} - \frac{(1-x^2)^{3/2}}{3}}$

#51 $x(p)$ $\frac{dx}{dp} = p\sqrt{p^2-25}$ $x(13) = 600$

$x = \int p\sqrt{p^2-25}\,dp$ $u = p^2-25$ $\frac{du}{dp} = 2p$ $x = \int \frac{1}{2}u^{1/2}\,du = \frac{\frac{1}{2}u^{3/2}}{3/2} + c = \frac{u^{3/2}}{3} + c$

$x(p) = \frac{(p^2-25)^{3/2}}{3} + c$ $x(13) = \frac{(144)^{3/2}}{3} + c = \frac{1728}{3} + c = 576 + c = 600$

$c = 58$

$\boxed{x(p) = \frac{(p^2-25)^{3/2}}{3} + 58}$

Sect 5.3

#7 $\int 5x^2 e^{x^3}\,dx$ $u = x^3$ $\frac{du}{dx} = 3x^2$ $\frac{5}{3}du = 5x^2\,dx$

$\int e^u \frac{5}{3}\,du = \frac{5}{3}e^u + c = \boxed{\frac{5}{3}e^{x^3} + c}$

#17 $\int \frac{2}{3x+5}\,dx$ $u = 3x+5$ $du = 3dx$ $\frac{2}{3}du = 2\,dx$

$\int \frac{2}{3}\frac{1}{u}\,du = \frac{2}{3}\ln u + c = \boxed{\frac{2}{3}\ln(3x+5) + c}$

#57 $\frac{dP}{dt} = \frac{3000}{1+0.25t}$ $P(0) = 1000$ $u = 1+0.25t$ $du = 0.25\,dt$ $4\,du = dt$

$P = \int \frac{3000}{1+0.25t}\,dt = 3000\int \frac{4}{u}\,du$

(a) $P(t) = 12{,}000\ln u + c = 12{,}000\ln(1+0.25t) + c$ $\boxed{P(t) = 12{,}000\ln(1+0.25t) + 1{,}000}$

$P(0) = c = 1000$

(b) $\boxed{P(3) = 7715}$

(c) $12000 = 12000\ln(1+0.25t) + 1000$

$\ln(1+0.25t) = \frac{11000}{12000} = 0.91666\ldots$ $1+0.25t = 2.500945$

$\boxed{t = 6.0038}$

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Sect 5.3

#61 $\frac{dS}{dt} = 1724.1\, e^{-\frac{t}{4.2}}$ $\quad S = 1724.1 \int e^{-\frac{t}{4.2}}\, dt$ $\quad u = -t/4.2$, $du = -1/4.2\, dt$, $-4.2\, du = dt$

$S(t) = 1724.1(-4.2)\, e^{-\frac{t}{4.2}} + c = -7241.2\, e^{-\frac{t}{4.2}} + c$

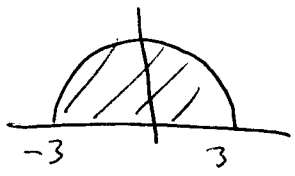
$S(5) = 40,520 = -7241.2(0.304076\ldots) + c \Rightarrow c = 42,722$

(a) $S(t) = 42,722 - 7241\, e^{-\frac{t}{4.2}}$

(b) $S(2) = 42,722 - 4498 = \boxed{38,224}$

Sect 5.4

#11 $\int_{-3}^{3} \sqrt{9-x^2}\, dx$



Semicircle of radius = 3 $\quad A = \frac{1}{2}\left[\pi (3)^2\right] = \boxed{4.5\pi}$

#31 $\int_{-1}^{1} (\sqrt[3]{t} - 2)\, dt = \int_{-1}^{1} t^{1/3} - 2\, dt = \left(\frac{3}{4} t^{4/3} - 2t\right)\Big|_{-1}^{1} = \left(\frac{3}{4} - 2\right) - \left(\frac{3}{4} + 2\right) = \boxed{-4}$

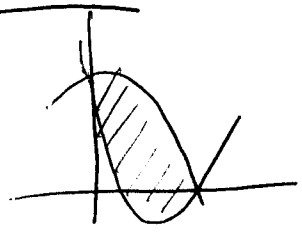
#35 $\int_{-1}^{0} t^{1/3} - t^{2/3}\, dt = \left(\frac{3}{4} t^{4/3} - \frac{3}{5} t^{5/3}\right)\Big|_{-1}^{0} = (0-0) - \left(\frac{3}{4} + \frac{3}{5}\right) = \boxed{\frac{-27}{20} = -1.35}$

#37 $\int_{0}^{4} \frac{1}{\sqrt{2x+1}}\, dx$ $\quad \left(\frac{d}{dx}(2x+1)^{1/2} = \frac{1}{2}(2x+1)^{-1/2} \cdot 2 = \frac{1}{\sqrt{2x+1}}\right)$

$= \left(\sqrt{2x+1}\right)\Big|_{0}^{4} = \sqrt{9} - \sqrt{1} = 3 - 1 = \boxed{2}$

Sect 5.5

#3



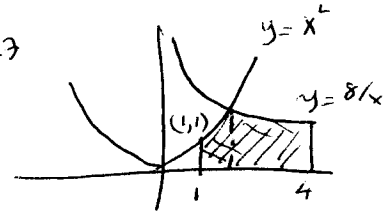
$g(x) = -x^2 + 2x + 3$
$f(x) = x^2 - 4x + 3$
(meet at $x = 0, 3$)

$A = \int_{0}^{3} (-x^2 + 2x + 3) - (x^2 - 4x + 3)\, dx$

$= \int_{0}^{3} (-2x^2 + 6x)\, dx = \left(-\frac{2}{3} x^3 + 3x^2\right)_{0}^{3}$

$= (-18 + 27) - (0 + 0) = \boxed{9}$

#27



$A = \int_{1}^{2} x^2\, dx + \int_{2}^{4} \frac{8}{x}\, dx = \frac{x^3}{3}\Big|_{1}^{2} + 8 \ln x\Big|_{2}^{4}$

$= \left(\frac{8}{3} - \frac{1}{3}\right) + (8 \ln 4 - 8 \ln 2) = 2.333\ldots + 5.545\ldots = \boxed{7.8785}$

or $\boxed{\frac{7}{3} + 8 \ln 2}$

Sect 5.5

#53 p = price, x = Demand

p	x
$325	6000
300	8000

Supply $P = 0.0275x$

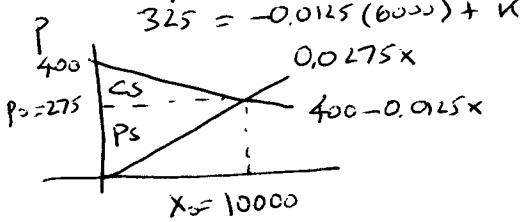
$m = \frac{300-325}{8000-6000} = \frac{-25}{2000} = -0.0125$

$P = -0.0125x + K$

$325 = -0.0125(6000) + K \Rightarrow K = 400$

$400 - 0.0125x = 0.0275x$

$400 = 0.04x \Rightarrow 10000 = x_0$



$C.S. = \int_0^{10000} (400 - 0.0125x) - 275 \, dx = \$625{,}000$

$P.S. = \text{area triangle} = \frac{10000(275)}{2} = 1{,}375{,}000$

Sect 5.6

#3 $h = \frac{1-0}{4} = 0.25$ $f(x) = \sqrt{x}$

$M_4 = 0.25[f(0.125) + f(0.375) + f(0.625) + f(0.875)]$

$= 0.25(2.6419\ldots) = \boxed{0.673\ldots}$

exact $\int_0^1 \sqrt{x}\, dx = \frac{x^{3/2}}{3/2}\Big|_0^1 = \frac{2}{3}(1-0) = \boxed{\frac{2}{3}}$

#12 $f(x) = x^2 - x^3$ on $[0,1]$ M_4 same as above

$M_4 = 0.25[0.01367 + 0.08789 + 0.14648 + 0.09570]$

$= 0.25\ [0.34374] = \boxed{0.08594}$

exact $\int_0^1 x^2 - x^3\, dx = \frac{x^3}{3} - \frac{x^4}{4} = \frac{1}{3} - \frac{1}{4} = \boxed{\frac{1}{12} = 0.08333\ldots}$

#27 $\int_0^2 x^3\, dx$ $T_8 = \frac{0.25}{2}[f(0) + 2f(0.25) + 2f(0.5) + f(0.75) + \ldots + 2f(1.75) + f(2)]$

$= \frac{1}{8}(32.5) = \boxed{4.0625}$

$M_8 = 0.25[f(0.125) + f(0.375) + f(0.625) + \ldots + f(1.875)]$

$= 0.25\ (15.875) = \boxed{3.96875}$ ← closer

exact $\frac{x^4}{4}\Big|_0^2 = \frac{16}{4} - \frac{0}{4} = \boxed{4}$