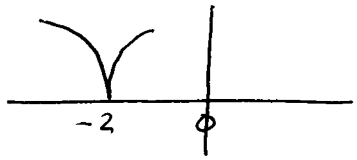
Math 109 - HW Answers - Chap 3 - Larson + Edwards 8th _Prof. R.B.Goldstein

Sect 3.1

#3 $f(x) = (x+2)^{2/3}$

$f'(x) = \frac{2}{3}(x+2)^{-1/3}$



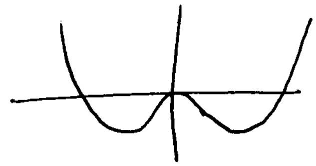
at (-2,0) the derivative does <u>not</u> exist - it is a <u>cusp</u> point

$f'(-3) = \frac{2}{3}(-1)^{-1/3} = \boxed{-\frac{2}{3}} < 0$, $f'(-1) = \frac{2}{3}(1)^{-1/3} = \boxed{\frac{2}{3}} > 0$

#7 $f(x) = x^4 - 2x^2$

$f'(x) = 4x^3 - 4x$

$= 4x(x-1)(x+1)$



$f'(x) < 0$ if $x < -1$

> 0 if $-1 < x < 0$

< 0 if $0 < x < 1$

> 0 if $1 < x$

or <u>decreasing</u> on $(-\infty, -1) \cup (0, 1)$; <u>increasing</u> $(-1, 0) \cup (1, \infty)$

#25 $y = 3x^3 + 12x^2 + 15x$

$y' = 9x^2 + 24x + 15 = 3(3x^2 + 8x + 5) = 3(3x+5)(x+1)$

$\Rightarrow \underline{x = -\frac{5}{3}}$ or $\underline{x = -1}$

$$\left.\begin{array}{l} f'(x) > 0 \text{ if } x < -\frac{5}{3} \\ < 0 \text{ if } -\frac{5}{3} < x < -1 \\ > 0 \text{ if } -1 < x \end{array}\right\} \Rightarrow \underline{\text{incr}}\ (-\infty, -\tfrac{5}{3}) \cup (-1, \infty) \quad \underline{\text{decr}}\ (-\tfrac{5}{3}, -1)$$

#43

(a) $$\left.\begin{array}{l} C = 0.6x + 7500 \quad 0 \le x \le 50{,}000 \\ R = \frac{1}{20000}(65{,}000x - x^2) \end{array}\right\} P = R - C = -\frac{x^2}{20{,}000} + 2.65x - 7500$$

(b) $P' = -\frac{x}{10{,}000} + 2.65 = 0 \Rightarrow x = 26{,}500$

$0 < x < 26{,}500$ $P' > 0$ incr

$26{,}500 < x < 50{,}000$ $P' < 0$ <u>decr</u>

(c) $x = 26{,}500 \Rightarrow C = \$23{,}400$, $R = \$51{,}012.50$, $P = \$27{,}612.50$

Sect 3.2

#9 $f(x) = x^3 - 6x^2 + 15$

$f'(x) = 3x^2 - 12x = 3x(x-4)$

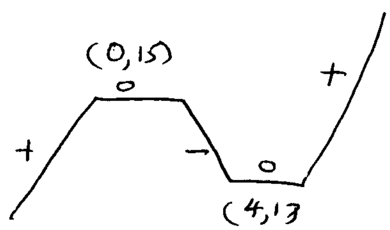
$f(0) = 15$ is a <u>maximum</u> pt.

$f(4) = -17$ is a <u>minimum</u> pt.

$f'(x) > 0$ if $x < 0$

< 0 if $0 < x < 4$

> 0 if $4 < x$



#15 $g(t) = t - \frac{1}{2t^2} = t - \frac{1}{2}t^{-2}$

$g'(t) = 1 - \frac{1}{2}(-2)t^{-3} = 1 + \frac{1}{t^3} = 0$

at $t = -1$ only real

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Sect 3.2

#25 $h(s) = \frac{1}{3-s} = (3-s)^{-1}$

$h'(s) = -(3-s)^{-2}(-1)$

$= \frac{1}{(3-s)^2} \neq 0 \Rightarrow$ monotonic on $[0,2]$

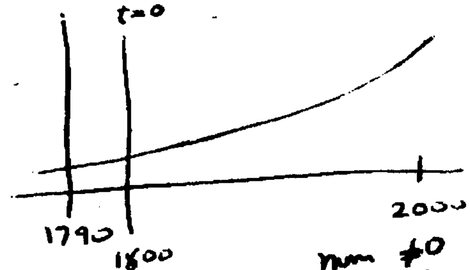
$\boxed{\begin{array}{ll} h(0) = \frac{1}{3-0} = \frac{1}{3} & \text{abs. min} \\ h(2) = \frac{1}{3-2} = 1 & \text{abs max} \end{array}}$

#51 $P = 0.00000583t^3 + 0.005003t^2 + 0.13775t + 4.658$ $\quad t = -10$ to 200 ($t=0$ 1800)

$P' = 0.00001749t^2 + 0.010006t + 0.13775$ (=0 only for $-14.1, -557.98$)

The graph is constantly increasing on this interval

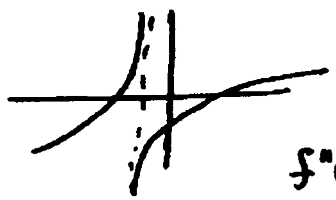
Min	1790	3.775 million
Max	2000	278.968 million



num ≠ 0

Sect 3.3

#3 $f(x) = \frac{x^2-1}{2x+1}$



$f'(x) = \frac{(2x+1)(2x) - (x^2-1)(2)}{(2x+1)^2} = \frac{2x^2+2x+2}{(2x+1)^2}$

$f''(x) = \frac{(2x+1)^2(4x+2) - (2x^2+2x+2)2(2x+1)(2)}{(2x+1)^4}$

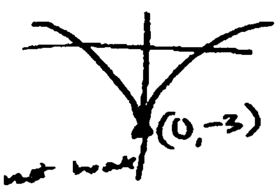
$f''(x) = \frac{(2x+1)(4x+2) - 4(2x^2+2x+2)}{(2x+1)^3} = -\frac{6}{(2x+1)^3} \neq 0$ for any x

concave up for $x < -\frac{1}{2}$; concave down for $x > -\frac{1}{2}$

#13 $f(x) = x^{2/3} - 3$

$f'(x) = \frac{2}{3}x^{-1/3} \neq 0$

2nd derivative does not work



$(0,-3)$ relative minimum

#31 $f(x) = x^3 - 9x^2 + 24x - 18$

$f'(x) = 3x^2 - 18x + 24$

$f''(x) = 6x - 18 = 0 \Rightarrow \boxed{x=3}$

$f''(x) < 0$ if $x < 3$
$\quad\quad > 0$ if $x > 3$

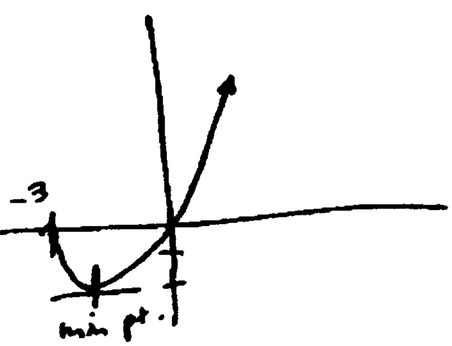
$\therefore$ $\boxed{f(3) = 54 \text{ is an infl. pt.}}$

#47 $g(x) = x\sqrt{x+3}$

$g'(x) = \sqrt{x+3} + x(\frac{1}{2})(x+3)^{-1/2} = \frac{3x+6}{2\sqrt{x+3}} = 0$

at $x=-2$ $\quad g'(-2^-) < 0$, $g'(-2^+) > 0$ min pt

(note: $g'(-3) = \infty$)



$g''(x) = \frac{3x+12}{4(x+3)^{3/2}} > 0$

for $x > -3$

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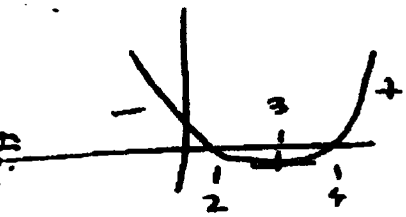
Sect 3.3

#51 $f(2)=0 \quad f(4)=0$

$f'(x)<0,\ x<3$ dec
$f'(3)=0$ $\Big\}$ min f
$f'(x)>0,\ x>3$ inc
$f''(x)>0 \ \forall x$ concave up

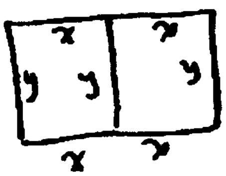


Sect 3.4

#5 $xy=192$, min $Q=x+y$ $\Big\}$ $y=192/x$, min $Q = x+192x^{-1}$

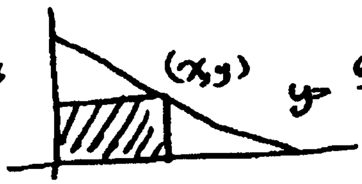
$\frac{d}{dx}(Q) = \frac{d}{dx}(x+192x^{-1}) = 1-\frac{192}{x^2}=0$
$\Rightarrow x^2=192 \Rightarrow x=\sqrt{192} = \underline{8\sqrt{3} = x}$
$Q = 8\sqrt{3}+8\sqrt{3} = \underline{16\sqrt{3}}$

#11



$4x+3y=200 \Rightarrow y=\frac{200-4x}{3}$
min $A=2xy = 2x\left(\frac{200-4x}{3}\right) = \frac{400}{3}x-\frac{8x^2}{3}$
$\frac{dA}{dx} = \frac{400}{3}-\frac{16x}{3}=0 \Rightarrow \underline{x=\frac{400}{16}=25}$ $\quad \underline{y=\frac{100}{3}}$ $\quad A=2(25)\left(\frac{100}{3}\right) = \underline{\frac{5000}{3}\text{ sq ft}}$

#23



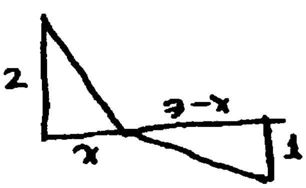
$y=\frac{6-x}{2}$ $\quad A=xy=x\left(\frac{6-x}{2}\right) = 3x-\frac{x^2}{2}$
$\frac{dA}{dx} = 3-x=0$ at $\underline{x=3,\ y=1.5}$ $\quad \underline{A=4.5}$

#27

$V=\pi r^2 h = 12$ fl oz $= 1.80469 \text{ in}^3 \Rightarrow h=\frac{1.80469}{\pi r^2} = 0.574451\, r^{-2}$
$A = 2\pi r^2 + \pi r^2 h = \pi(2r^2+rh) = \pi(2r^2+0.574451\, r^{-1})$
$\frac{dA}{dr} = \pi(4r-0.574451\, r^{-2})=0 \Rightarrow r=\left(\frac{0.574451}{4}\right)^{1/3} = \underline{0.523678}$

$\underline{h=2.09471}$ (note $h=4r=2d$)

#38*



time $=\frac{\text{distance}}{\text{rate}}$ $\quad$ Let T = total time

$T=\frac{\sqrt{4+x^2}}{2}+\frac{\sqrt{1+(3-x)^2}}{4} = \frac{1}{2}(4+x^2)^{1/2}+\frac{1}{4}(x^2-6x+10)^{1/2}$

$\frac{dT}{dx} = \frac{\frac{1}{2}(4+x^2)^{-1/2}(2x)}{2} + \frac{1}{4}\cdot\frac{1}{2}(x^2-6x+10)^{-1/2}(2x-6) = 0$

$\frac{x}{2\sqrt{4+x^2}}+\frac{x-3}{4\sqrt{x^2-6x+10}}=0 \Rightarrow \frac{x^2}{\cancel{4}(4+x^2)} = \frac{x^2-6x+9}{\cancel{16}\,4(x^2-6x+10)}$

$\Rightarrow 4x^4-24x^3+40x^2 = x^4-6x^3+13x^2-24x+36$
$\Rightarrow 3x^4-18x^3+27x^2+24x-36 = \underline{3(x^4-6x^3+9x^2+8x-12)=0}$

roots are $x=1$ (answer), $\underbrace{x=-1.116\ldots,\ 3.058\ldots,\ \pm 1.2067i}_{\text{reject these roots}}$

* too difficult for exam

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Sect 3.5

#3 $R = 400x - x^2$

$R' = 400 - 2x = 0 \Rightarrow x = 200$ units

#7 $C = 2x^2 + 255x + 5000$

$\bar{C} = 2x + 255 + 5000x^{-1}$ $\quad \bar{C}' = 2 - 5000x^{-2} = 0 \Rightarrow x^2 = 2500 \Rightarrow \underline{x = 50}$

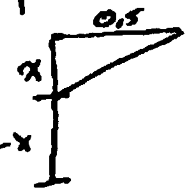
#11 $C = 8000 + 50x + 0.03x^2 \quad p = 70 - 0.001x$

$P = R - C = xp - C = 70x - 0.001x^2 - (8000 + 50x + 0.03x^2)$

$P = -0.031x^2 + 20x - 8000$

$P' = -0.062x + 20 = 0 \quad \Rightarrow \quad x = \frac{20}{0.062} = 322.58$

$p = 70 - 0.32258 = \underline{\$69.68}$

#23 

$C = 8\sqrt{x^2 + 0.5^2} + 6(6 - x) = 8(x^2 + 0.25)^{1/2} + 6(6-x)$

$C' = 8\left(\frac{1}{2}\right)(x^2 + 0.25)^{-1/2}(2x) - 6 = 0$

$\frac{\cancel{8}^{4} x}{\sqrt{x^2 + 0.25}} = \cancel{6}^{3} \Rightarrow \quad 16x^2 = 9(x^2 + 0.25) = 9x^2 + 2.25$

$7x^2 = 2.25 \quad x = \left(\frac{2.25}{7}\right)^{1/2} \approx \underline{0.567}$

#30 $p = 20 - 0.0002x, \; x = 30 \quad p(30) = 20 - 0.006 = 19.994$

$\frac{dp}{dx} = -0.0002\Big|_{x=30} = -0.0002 \quad \eta = \frac{19.994/30}{-0.0002} = \underline{\frac{-9997}{3}}$ $\quad$ elastic

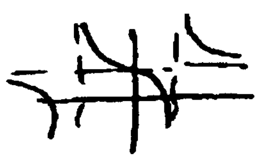
$\eta = \frac{(20 - 0.0002x)/x}{-0.0002} = -\frac{100{,}000}{x} + 1 = -1 \qquad 2 = \frac{100000}{x}$

$x = \frac{100000}{2} = 50000$

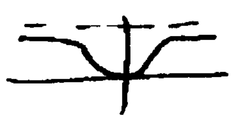
elastic $(0, 50000)$

inelastic $(50000, 100000)$

Sect 3.6

#3 $f(x) = \frac{x^2 - 2}{x^2 - x - 2} = \frac{x^2 - 2}{(x+1)(x-2)}$ 

Vertical $\underline{x = -1, \; x = 2}$ $\qquad$ Horizontal $\lim\limits_{x \to \infty} \frac{x^2 - 2}{x^2 - x - 2} = \underline{1 = y}$

#5 $f(x) = \frac{3x^2}{2(x^2+1)}$  $\quad \lim\limits_{x \to \infty} \frac{3x^2}{2(x^2+1)} = \underline{\frac{3}{2} = y}$ $\quad$ Horizontal only

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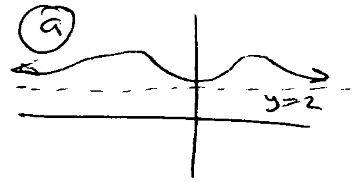
Sect 3.6

#11 $f(x) = 2 + \dfrac{x^2}{x^4+1}$ horiz. asymp. at $y=2$

(a)

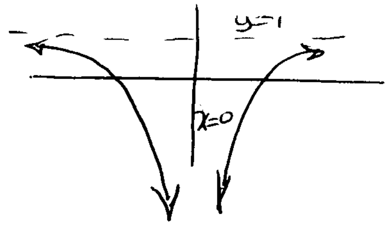


#51 $y = 1 - 3x^{-2}$ intercepts $(\pm\sqrt{3}, 0)$

$y' = \dfrac{6}{x^3} \neq 0$ horiz. asymp. $y=1$

vertical $x=0$

no relative max or min

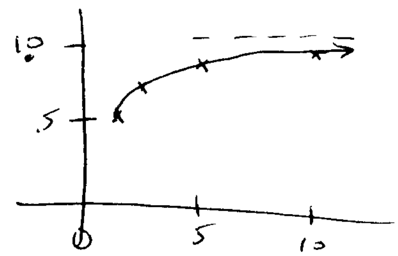


#65 $P = \dfrac{0.5 + 0.9(n-1)}{1 + 0.9(n-1)}$

(a)

n	1	2	3 … 5	… 8		10
P	0.5	0.737	0.821	0.891	0.932	0.945

as $n \to \infty$ $P \to \dfrac{0.9}{0.9} = 1$



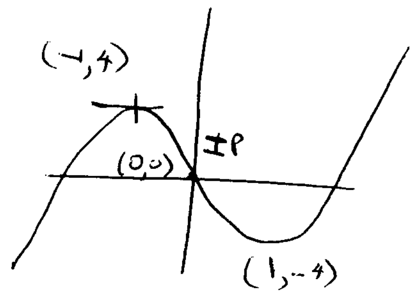
Sect 3.7

#17 $y = x^5 - 5x$ $y(0)=0$

$y' = 5x^4 - 5 = 5(x^4-1) = 5(x-1)(x+1)(x^2+1) = 0$ at $x = 1, -1$

$y'' = 20x^3 = 0$ at $x=0$ only, $y''(1) = 20 > 0$

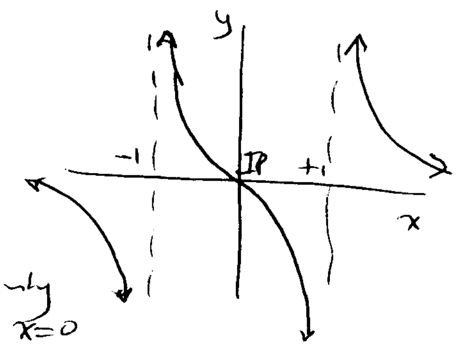
$y''(-1) = -20 < 0$



#37 $y = \dfrac{2x}{x^2-1}$ $y(0)=0$

$y' = \dfrac{(x^2-1)2 - 2x(2x)}{(x^2-1)^2} = \dfrac{-2x^2-2}{(x^2-1)^2} < 0 \quad \forall x$ horiz. asymp $y=0$

$y'' = \dfrac{(x^2-1)^2(-4x) - (-2x^2-2)2(x^2-1)2x}{(x^2-1)^4} = \dfrac{8x^3+4x}{(x^2-1)^3} = 0$ only at $x=0$



to see asymptotes consider $x = -0.9, 0.9$
$-1.1, -5, \ldots, +1.1, 5$

Sect 3.8

#23 $f(x) = \dfrac{x}{x^2+1}$ $(0,0)$

$f'(x) = \dfrac{(x^2+1) - x(2x)}{(x^2+1)^2} = \dfrac{1-x^2}{(x^2+1)^2}$

$f'(0) = 1$

$y - 0 = 1(x-0) \Rightarrow y = x$ is tangent line

$f(0+0.01) = 0.009999\ldots$
$y(0.01) = 0.01$

$f(0-0.01) = -0.009999\ldots$
$y(-0.01) = -0.01$

Sect 3.8

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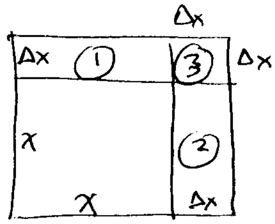
#33 $P = -0.5x^3 + 2500x - 6000 \quad x = 50$

$\left.\begin{array}{l} P(50) = 56{,}500 \\ P(51) = 55{,}174.5 \end{array}\right\} \Rightarrow \Delta P = \underline{\underline{-1325.5}}$

$P' = -1.5x^2 + 2500\Big|_{50} = \underline{\underline{-1250}}$



#39 $A = x^2$

$dA = 2x\,dx \simeq 2x\,\Delta x$

$A(x+\Delta x) - A(x) = (x+\Delta x)^2 - x^2 = x^2 + 2x\,\Delta x + (\Delta x)^2 - x^2 = 2x\,\Delta x + (\Delta x)^2$

$dA = 1 + 2$ extra region $(\Delta x)^2$

$\Delta A = 1 + 2 + 3$

$dA = 2(12)\left(\pm\frac{1}{64}\right) = \boxed{\pm\frac{3}{8}\ in^2}$

When $A = 144$ the relative error is $\frac{dA}{A} = \frac{\pm 3/8}{144} = \underline{\underline{0.0026}}$

#43

$B = 0.1\sqrt{5w} = 0.1\sqrt{5}\,w^{1/2}$

$\frac{dB}{dw} = 0.1\sqrt{5}\left(\frac{1}{2}\right)w^{-1/2}$

$\Delta B \approx dB = \frac{0.1\sqrt{5}}{2\sqrt{w}}\,dw\Big|_{\substack{w=90 \\ dw=5}} = \frac{0.1\sqrt{5}(5)}{2\sqrt{90}} = \underline{\underline{0.0589\ m^2}}$