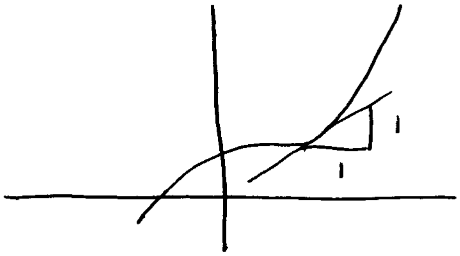
Math 109 - Prof. Richard B. Goldstein - HW Answers - Larson 8th

Chapter 2

Sect 2.1

#5



$m = \frac{1}{1} = 1$

#19 $f(x) = x^2 - 1 \quad (2,3)$

$$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0} \frac{[(2+h)^2 - 1] - 3}{h}$$

$$= \lim_{h\to 0} \frac{h^2 + 4h + \cancel{3} - \cancel{3}}{h} = \lim_{h\to 0} \frac{h^2+4h}{h}$$

$$= \lim_{h\to 0} h + 4 = \boxed{4}$$

#33 $h(t) = \sqrt{t-1}$

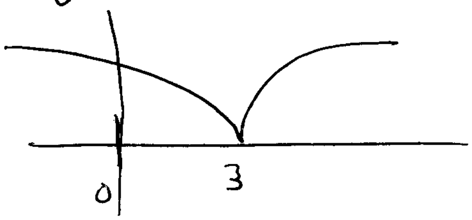
$$\frac{h(t+\Delta t) - h(t)}{\Delta t} = \frac{\sqrt{t+\Delta t - 1} - \sqrt{t-1}}{\Delta t}$$

$$= \frac{\sqrt{t+\Delta t-1} - \sqrt{t-1}}{\Delta t} \cdot \frac{\sqrt{t+\Delta t-1} + \sqrt{t-1}}{\sqrt{t+\Delta t-1} + \sqrt{t-1}}$$

$$= \frac{(t+\Delta t - 1) - (t-1)}{\Delta t(\sqrt{t+\Delta t-1} + \sqrt{t-1})} = \frac{\Delta t}{\Delta t(\sqrt{t+\Delta t-1} + \sqrt{t-1})}$$

$$\lim_{\Delta t\to 0} \frac{h(t+\Delta t) - h(t)}{\Delta t} = \lim_{\Delta t\to 0} \frac{1}{\sqrt{t+\Delta t-1} + \sqrt{t-1}} = \frac{1}{2\sqrt{t-1}}$$

#53 $y = (x-3)^{2/3}$



differentiable for $x \neq 3$ $\quad (-\infty, +3) \cup (+3, \infty)$

cusp at $(3,0)$

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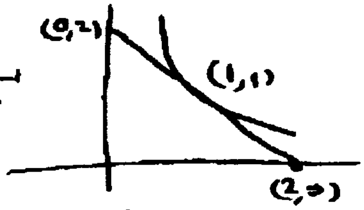
Sect 2.2

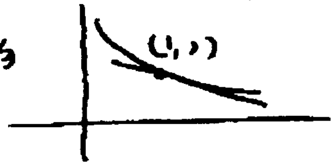
#3 (a) $y = x^{-1}$



$y' = -1(x^{-2})\Big|_{x=1} = -1$

$\boxed{m = -1}$

(b) $y = x^{-1/3}$



$y' = -\frac{1}{3}(x^{-4/3})\Big|_{x=1} = -\frac{1}{3}$

$\boxed{m = -1/3}$

#21 $y = 4x^{-2} + 2x^{2}$

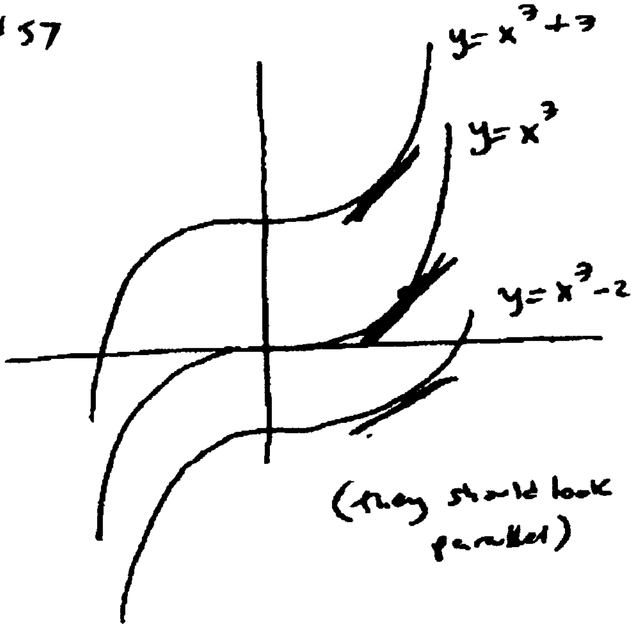
$y' = 4(-2)x^{-3} + 2(2)x^{1} = \boxed{-8x^{-3} + 4x}$ or $\boxed{-\frac{8}{x^3} + 4x}$

#25 $y = \frac{1}{(4x)^3} = \frac{1}{64}x^{-3}$ $\quad y' = -\frac{3}{64}x^{-4}$ or $\boxed{-\frac{3}{64x^4}}$

#39 $f(x) = x(x^2+1) = x^3 + x$

$f'(x) = \boxed{3x^2 + 1}$

#57



(they should look parallel)

$y' = 3x^2$ for all 3 cases

$m = y'(1) = 3(1)^2 = 3$

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Sect 2.3

\$ billion

#1 (a) 1980–1985: $\frac{115-63}{5} = \frac{52}{5} = 10.4$ per yr.

(b) 1985–1990: $\frac{152-115}{5} = \frac{37}{5} = 7.4$ per yr.

(c) 1990–1995: $\frac{184-152}{5} = \frac{32}{5} = 6.4$ per yr.

(d) 1995–2000: $\frac{267-184}{5} = \frac{82}{5} = 16.4$ per yr.

(e) 1980–2004: $\frac{312-63}{24} = \frac{249}{24} = 10.375$ per yr.

(f) 1990–2004: $\frac{312-152}{14} = \frac{160}{14} = 11.429$ per yr.

#12 $g(x) = x^3 - 1$; $[-1, 1]$

$g(-1) = -1-1 = -2$
$g(1) = 1-1 = 0$

$\frac{0-(-2)}{1-(-1)} = \frac{2}{2} = 1$ avg. rate

$g'(x) = 3x^2$ $\quad g'(-1) = g'(1) = 3$

#17 $S = -16t^2 + 555$

(a) $S(2) = 491$, $S(3) = 411$ $\quad \frac{S(3)-S(2)}{3-2} = \frac{411-491}{1} = -80$ ft/sec

(b) $V = \frac{dS}{dt} = -32t$ $\quad V(2) = -64$ ft/sec $\quad V(3) = -96$ ft/sec

(c) $S = 0$ at $t = \sqrt{\frac{555}{16}} \cong 5.89$ sec $\quad$ (d) $V(5.89) = -32(5.89) \approx -188.5$ ft/sec

#32 $R = 2x(900 + 32x - x^2) = 1800x + 64x^2 - 2x^3$

(a) $R(15) - R(14) = 34{,}650 - 32{,}256 = 2{,}394$

(b) $R' = 1800 + 128x - 6x^2 \Big|_{x=14} = 2{,}416$

(c) relatively close

#44 $C = \frac{1{,}008{,}000}{Q} + 6.3Q$

$Q = 351 \Rightarrow C = 5083.09$
$Q = 350 \Rightarrow C = 5085$

$5083.09 - 5085 = -\$1.91$

$\frac{dC}{dQ} = -\frac{1{,}008{,}000}{Q^2} + 6.3 \Big|_{350} = -\$1.93$

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Sec 2.4

#7 $g(x) = (x^2-4x+3)(x-2)$ at $(4,6)$

$$g'(x) = (2x-4)(x-2) + (x^2-4x+3)\cdot 1 \Big|_{x=4} = 4\cdot 2 + 3\cdot 1 = \underline{\underline{11}}$$

#11 $f(t) = \dfrac{2t^2-3}{3t+1}$ at $\left(3, \frac{3}{2}\right)$

$$f'(t) = \frac{(3t+1)(4t) - (2t^2-3)(3t+1)}{(3t+1)^2} = \frac{6t^2+4t+9}{(3t+1)^2}\Big|_3 = \frac{75}{100} \Rightarrow \boxed{\frac{3}{4}}$$

#59 $P = 500\left(1 + \dfrac{4t}{50+t^2}\right)$

$$P' = 500\left(0 + \frac{(50+t^2)(4) - (4t)(2t)}{(50+t^2)^2}\right) = 500\left(\frac{200-4t^2}{(50+t^2)^2}\right)\Big|_{t=2}$$

$$= 500\left(\frac{200-16}{(54)^2}\right) = \quad 31.55 \text{ bacteria/hr}$$

#61 Demand $\propto \dfrac{1}{\text{price}^2}$ $\qquad x = \dfrac{k}{p^2}$ for $x \geq 5$

(a) $x = 16 = \dfrac{k}{(1000)^2} \Rightarrow k = 16{,}000{,}000$

$$\boxed{x = \frac{16{,}000{,}000}{p^2}} \quad \text{or} \quad p = \sqrt{\frac{16{,}000{,}000}{x}} = \frac{4{,}000}{\sqrt{x}}$$

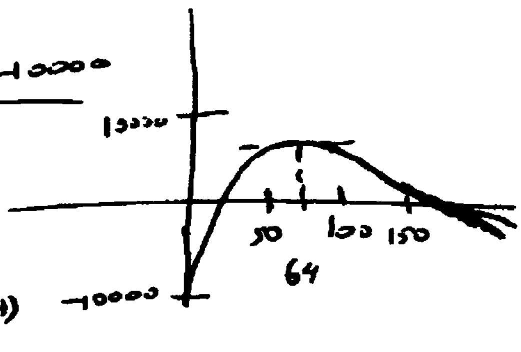
(b) cost = \$250 per item
fixed cost = 10000 $\Big\} \Rightarrow \boxed{\text{Cost} = 250x + 10{,}000}$

(c) Revenue $= xp \Rightarrow R = x\dfrac{4000}{\sqrt{x}} = 4000\sqrt{x} = 4000x^{1/2}$

Profit $= R - C = \underline{4000x^{1/2} - 250x - 10000}$

$$P' = \frac{2000}{\sqrt{x}} - 250 = 0 \text{ at } \sqrt{x} = 8$$
$$x = 64$$

$$p = \frac{4000}{\sqrt{64}} = \frac{4000}{8} = \underline{\underline{\$500}} \text{ (peak profit)}$$



$\underline{P(64) = \$6{,}000}$

Sect 2.5

#7 $y = (3x+1)^{-1}$ $\quad \underline{u = 3x+1}$ $\quad \underline{y = f(u) = u^{-1}}$

#39 $h(x) = (4-x^3)^{-4/3}$ $\quad h'(x) = -\frac{4}{3}(4-x^3)^{-7/3}(-3x^2)$
$= 4x^2(4-x^3)^{-7/3} = \boxed{\dfrac{4x^2}{(4-x^3)^{7/3}}}$

#59 $f(x) = x(3x-9)^3$
$f'(x) = 1\cdot(3x-9)^3 + x\cdot 3(3x-9)^2(3) = \underline{(3x-9)^3 + 9x(3x-9)^2}$

#61 $y = x\sqrt{2x+3} = x(2x+3)^{1/2}$
$y' = 1\cdot\sqrt{2x+3} + x\cdot\frac{1}{2}(2x+3)^{-1/2}\cdot 2 = \boxed{\sqrt{2x+3} + \dfrac{x}{\sqrt{2x+3}}}$

or $\dfrac{2x+3 \;+ x}{\sqrt{2x+3}} = \boxed{\dfrac{3(x+1)}{\sqrt{2x+3}}}$

Sect 2.6

#25 $f(x) = \sqrt{4-x} = (4-x)^{1/2}$
$f'(x) = \frac{1}{2}(4-x)^{-1/2}\cdot(-1) = -\frac{1}{2}(4-x)^{-1/2}$
$f''(x) = -\frac{1}{2}\left(-\frac{1}{2}\right)(4-x)^{-3/2}(-1) = \frac{-1}{4}(4-x)^{-3/2}$
$f'''(x) = \frac{-1}{4}\left(-\frac{3}{2}\right)(4-x)^{-5/2}(-1) = -\frac{3}{8}(4-x)^{-5/2}$
$f'''(-5) = -\frac{3}{8}(9)^{-5/2} = -\frac{3}{8}(3)^{-5} = -\frac{3}{8\cdot 243} = \boxed{-\frac{1}{648}}$

#41 $f(x) = \dfrac{x}{x^2+3}$ $\quad f'(x) = \dfrac{(x^2+3)\cdot 1 - x(2x)}{(x^2+3)^2} = \dfrac{3-x^2}{(x^2+3)^2}$

$f''(x) = \dfrac{(x^2+3)^2(-2x) - (3-x^2)\,2(x^2+3)\,2x}{(x^2+3)^4} = \dfrac{(x^2+3)(-2x) - (3-x^2)4x}{(x^2+3)^3}$

$= \boxed{\dfrac{2x^3-18x}{(x^2+3)^3}}$ $\quad 2x^3 - 18x = 0$
$2x(x^2-9) = 0 \Rightarrow \boxed{x = -3, 0, +3}$
$2x(x-3)(x+3) = 0$

Sect 2.7

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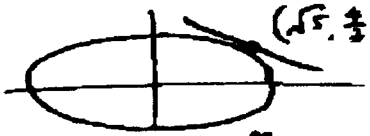


#29 $4x^2+9y^2=36$

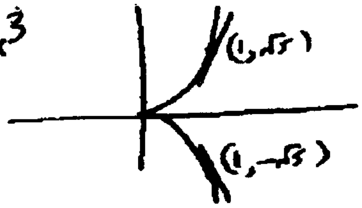
$8x+18yy'=0 \Rightarrow y' = -\frac{8x}{18y} = -\frac{4x}{9y}\Big|_{(\sqrt{5},\frac{4}{3})}$

$= -\frac{4\sqrt{5}}{9(4/3)} = \boxed{-\frac{\sqrt{5}}{3}}$

#21 $x^{1/2}+y^{1/2}=9$ pt. $(16,25)$ $\quad \frac{1}{2}x^{-1/2}+\frac{1}{2}y^{-1/2}y'=0$

$y' = \frac{-\frac{1}{2}x^{-1/2}}{\frac{1}{2}y^{-1/2}} = -\frac{\sqrt{y}}{\sqrt{x}}\Big|_{(16,25)} = -\frac{\sqrt{25}}{\sqrt{16}} = \boxed{-\frac{5}{4}}$

#37 $y^2=5x^3$



$2yy'=15x^2 \Rightarrow y' = \frac{15x^2}{2y}$

at $(1,\sqrt{5})$ $y' = \frac{15(1)^2}{2\sqrt{5}} = \boxed{\frac{3\sqrt{5}}{2}}$

at $(1,-\sqrt{5})$ $y' = \frac{15(1)^2}{-2(\sqrt{5})} = \boxed{-\frac{3\sqrt{5}}{2}}$

tangent line $y-\sqrt{5} = \frac{3\sqrt{5}}{2}(x-1) \Rightarrow \sqrt{5}y-5 = \frac{15}{2}x - \frac{15}{2}$

$2\sqrt{5}y-10 = 15x-15$

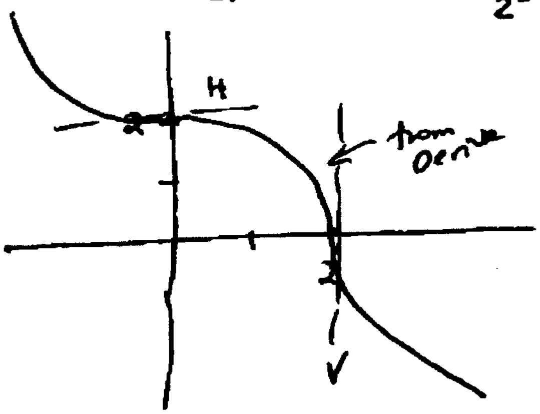
$\underline{\underline{15x-2\sqrt{5}y-5=0}}$

similarly the other tangent line is $\underline{\underline{15x+2\sqrt{5}y-5=0}}$

#39 $x^3+y^3=8$ $\quad (0,2)$ and $(2,0)$

$3x^2+3y^2y'=0 \Rightarrow y' = -\frac{3x^2}{3y^2} = -\frac{x^2}{y^2}$

at $(0,2)$ $m = -\frac{0}{2^2} = 0$ $\quad y-2 = 0(x-0) \Rightarrow \boxed{y=2}$



for $(2,0)$ $\underline{m=\infty}$ $\therefore$ vertical line

$x-2=0$ $\quad \boxed{x=2}$

Sect 2.8

#1 $y=\sqrt{x}$ $\qquad \frac{dy}{dt}=\frac{1}{2\sqrt{x}}\frac{dx}{dt}$

or (a) $\dot y = \left(\frac{1}{2\sqrt{x}}\right)\dot x \Big|_{x=4,\ \dot x=3} = \frac{1}{2\sqrt4}3 = \frac{3}{4}$

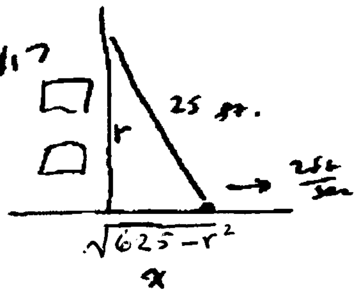
(b) $2=\frac{1}{2\sqrt{25}}\dot x \qquad \dot x = 20 \qquad = \boxed{62}$

#9 $V=\frac{4}{3}\pi r^3 \qquad \dot V = 10\ \frac{ft^3}{min} \qquad \dot r = ?$ when $r = 1, 2$ ft.

$\dot V = \frac{4}{3}\pi\, 3r^2 \dot r \qquad \dot r = \frac{\dot V}{4\pi r^2}$ at $r=1$ $\quad \frac{10}{4\pi(1)^2} = \frac{5}{2\pi} = \underline{\underline{0.80\ ft/min}}$

at $r=2$ $\quad \frac{10}{4\pi(2)^2} = \frac{5}{8\pi} = \underline{\underline{0.20\ ft/min}}$

#17



$x^2+r^2 = 25^2 = 625$

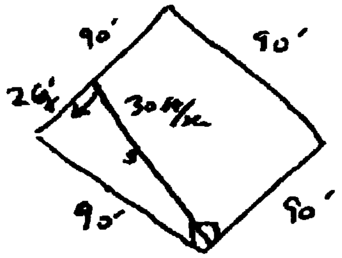
$2x\dot x + 2r\dot r = 0 \qquad \dot r = -\frac{x}{r}\dot x$

(a) $x=7 \Rightarrow r=24 \qquad \dot r = -\frac{7}{24}(2)\ ft/sec = \boxed{-\frac{7}{12}\ ft/sec}$

(b) $x=15 \Rightarrow r=20 \qquad \dot r = -\frac{15}{20}(2) = -0.75(2)\ ft/sec = \boxed{-1.5\ ft/sec}$

(c) $x=24 \Rightarrow r=7 \qquad \dot r = -\frac{24}{7}(2)\ ft/sec = \boxed{-\frac{48}{7}\ ft/sec}$

#21



$x^2+90^2=s^2 \qquad \dot s = \frac{x\dot x}{s} \qquad \dot x = -30\ ft/sec$

$2x\dot x = 2s\dot s$

is $x=26$, $s=\sqrt{26^2+90^2} = \sqrt{8776} = 93.68$ ft

$= \frac{26(-30)}{93.68} = \underline{\underline{-8.326\ ft/sec}}$

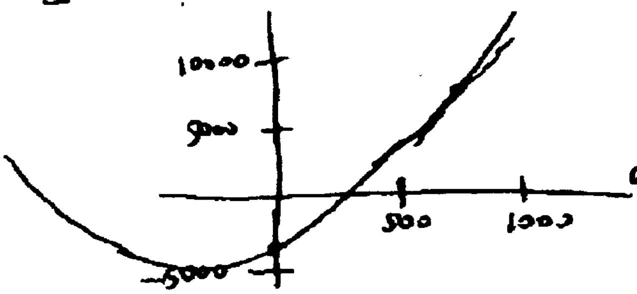
#24 $\dot x = 25\ \frac{units}{wk} \qquad x = 800$

$p = 50 - 0.01x \qquad \Rightarrow R = xp = 50x - 0.01x^2$

$C = 4000 + 40x - 0.02x^2$

$P = 50x - 0.01x^2 - (4000+40x-0.02x^2) = 0.01x^2 + 10x - 4000$

$\dot P = 0.02x\dot x + 10\dot x = (0.02x+10)\dot x \Big|_{\dot x = 25,\ x=800} = \boxed{\$650\ /wk}$



note: $P(800) = \$10{,}400$
$P(825) = \$11{,}056.25$

after 1 wk

$P(825)-P(800) = \boxed{\$656.25}$