Math 110 – Prof. Richard B. Goldstein – Larson 8th Ed – Chap 10 HW

Sect 10.1

#7 $a_n = \frac{3^n}{n!}$ $a_1 = \frac{3^1}{1!} = \boxed{3}$, $a_2 = \frac{3^2}{2!} = \boxed{\frac{9}{2}}$, $a_3 = \frac{3^3}{3!} = \frac{27}{6} = \boxed{\frac{9}{2}}$, $a_4 = \frac{3^4}{4!} = \frac{81}{24} = \boxed{\frac{27}{8}}$, $a_5 = \boxed{\frac{81}{40}}$

#15 $a_n = \frac{n^2+3n-4}{2n^2+n-3}$ $\lim_{n\to\infty} \frac{n^2+3n-4}{2n^2+n-3} \overset{L'H}{=} \lim_{n\to\infty} \frac{2n+3}{4n+1} \overset{L'H}{=} \lim_{n\to\infty} \frac{2}{4} = \boxed{\frac{1}{2}}$

#31 $a_n = (-1)^n\left(\frac{n}{n+1}\right)$ $\lim_{n\to\infty} \frac{n}{n+1} = \lim_{n\to\infty} \frac{1}{1+\frac{1}{n}} = 1$ $\therefore$ alternates $-1, +1, -1, +1, \ldots$

$\boxed{\text{does not converge}}$

#41 $2, -1, \frac{1}{2}, -\frac{1}{4}, 8, \ldots$ $\boxed{a_n = \frac{(-4)}{(-2)^n} \text{ or } (-4)(-2)^{-n}}$

#59 $\frac{8}{3}, \frac{10}{3}, 4, \frac{14}{3}, \ldots = \frac{8}{3}, \frac{10}{3}, \frac{12}{3}, \frac{14}{3}, \ldots$ Common difference $d = \frac{2}{3}$ $\boxed{\text{arithmetic}}$

Sect 10.2

#9 $\sum_{n=1}^{\infty} \frac{n}{n+1} = \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \ldots$ $\boxed{\text{diverges}}$ since $a_n \to 1$ (not 0)

#13 $\sum_{n=0}^{\infty} 2\left(\frac{3}{4}\right)^n = 2 + \frac{3}{2} + \frac{9}{8} + \frac{27}{32} + \frac{81}{128} + \ldots$ $|r| = \frac{3}{4} < 1$ $\therefore$ $\boxed{\text{converges}}$

note sum $= \frac{a}{1-r} = \frac{2}{1-3/4} = \frac{2}{1/4} = 8$

#27 $\sum_{n=0}^{\infty}\left(\frac{1}{2^n} - \frac{1}{3^n}\right) = \sum_{n=0}^{\infty} \frac{1}{2^n} - \sum_{n=0}^{\infty} \frac{1}{3^n}$ $\frac{1}{1-\frac{1}{2}} - \frac{1}{1-\frac{1}{3}} = 2 - \frac{3}{2} = \boxed{\frac{1}{2}}$

#49 $A = 100\left(1 + \frac{0.10}{12}\right) + \ldots + 100\left(1+\frac{0.10}{12}\right)^{60}$

$\frac{A}{1+\frac{0.10}{12}} = 100 + \ldots + 100\left(1+\frac{0.10}{12}\right)^{59}$

$A - \frac{A}{1+\frac{0.10}{12}} = 100\left(1+\frac{0.10}{12}\right)^{60} - 100$

$0.008264463 A = 64.530893$

$\boxed{A = \$7{,}808.24}$

Sect 10.3

#9 $\sum_{n=1}^{\infty} \frac{1}{n^3}$ $p = 3 > 1$ $\boxed{\text{Converges}}$

#17 $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \ldots = \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ $p = \frac{3}{2} > 1$ $\boxed{\text{converges}}$

#23 $\sum_{n=1}^{\infty} \frac{n}{4^n}$ $\frac{a_{n+1}}{a_n} = \frac{\frac{n+1}{4^{n+1}}}{\frac{n}{4^n}} = \frac{n+1}{4n} \longrightarrow \frac{1}{4}$ as $n\to\infty$ $\frac{1}{4} < 1$ $\therefore$ $\boxed{\text{converges}}$

#37 $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ same as #17 $p = \frac{3}{2} > 1$ $\boxed{\text{converges}}$

$\sum \frac{1}{n^2} = \pi^2/6$ $\sum \frac{1}{n^3} = \zeta(3) \approx 1.20206$

#55 $\sum_{n=1}^{\infty}\left(\frac{1}{n^2} - \frac{1}{n^3}\right) = \sum_{n=1}^{\infty} \frac{1}{n^2} - \sum_{n=1}^{\infty} \frac{1}{n^3}$ $\left.\begin{matrix} p = 2 > 1 \\ p = 3 > 1 \end{matrix}\right\}$ both $\boxed{\text{converge}}$ difference $\approx \boxed{0.442877}$

Sect 10.4

#3 $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x+1)^n}{n!} = \boxed{1 - \frac{x+1}{1} + \frac{(x+1)^2}{2} - \frac{(x+1)^3}{6} - \frac{(x+1)^4}{24} + \ldots}$

#17 $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-1)^{n+1}}{n+1}$

$$\left| \frac{\dfrac{(-1)^{n+2}(x-1)^{n+2}}{n+2}}{\dfrac{(-1)^{n+1}(x-1)^{n+1}}{n+1}} \right| = \left|(x-1)\frac{n+1}{n+2}\right| \longrightarrow \boxed{|x-1|<1}$$

radius $=1$ center $=1$

#23 $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$

$$\left| \frac{\dfrac{x^{2n+3}}{(2n+3)!}}{\dfrac{x^{2n+1}}{(2n+1)!}} \right| = \left| \frac{x^2}{(2n+3)(2n+2)} \right| \longrightarrow 0 \quad \forall x \quad \therefore \boxed{\text{radius} = \infty}$$

#27 $f(x) = e^{3x}$ $c=0$

$f(0)=1$
$f'(x) = 3e^{3x}$, $f'(0)=3$
$f''(x) = 9e^{3x}$, $f''(0)=9$
$f'''(x) = 27e^{3x}$, $f'''(0)=27$

$$\boxed{1 + 3x + \frac{3^2x^2}{2!} + \frac{3^3x^3}{3!} + \ldots + \frac{3^n x^n}{n!} + \ldots}$$

#35 $f(x) = \dfrac{1}{\sqrt{1-x}} = (1-x)^{-1/2}$ $k = -\frac{1}{2}$ (see p699)

$$= 1 + \left(-\tfrac{1}{2}\right)(-x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(-x)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}(-x)^3 + \ldots$$

converges for $|x|<1$ $\boxed{1 + \frac{x}{2} + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \ldots}$ $R=1$

Sect 10.5

#5 $f(x) = \ln(x+1) = \boxed{x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots}$

$S_1(x) = x$, $S_2(x) = x - \frac{x^2}{2}$, $S_3(x) = x - \frac{x^2}{2} + \frac{x^3}{3}$, $S_4(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$

#11 $f(x) = \dfrac{x}{x+1} = \dfrac{x}{1+x}$ $a = x$ $r = -x$ geometric series

$= \boxed{x - x^2 + x^3 - x^4 + \ldots}$

$S_1(x) = x$, $S_2(x) = x - x^2$, $S_3(x) = x - x^2 + x^3$
$S_4(x) = x - x^2 + x^3 - x^4$

#29 $f(x) = \dfrac{1}{\sqrt{1+x^2}}$ (see p699) $k = -\frac{1}{2}$

$$= 1 + \left(-\tfrac{1}{2}\right)x^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(x^2)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}(x^2)^3$$

$$f(x) = 1 - \frac{x^2}{2} + \frac{3x^4}{8} - \frac{5}{16}x^6 + \ldots$$

$$\int_0^{1/2} f(x)\,dx = x - \frac{x^3}{6} + \frac{3x^5}{40} - \frac{5x^7}{112} + \ldots \Big|_0^{1/2} = 0.50000 - 0.02083 + 0.00234 - 0.00035$$

$\boxed{0.4812}$

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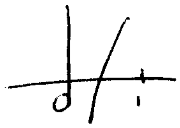
Sect 10.6

#7 $f(x) = \ln x + x$



$f'(x) = \frac{1}{x} + 1$

$$x_{n+1} = x_n - \frac{(\ln x_n) + x_n}{\frac{1}{x_n} + 1}$$

let $x_0 = 1$ $\quad x_1 = 1 - \frac{0+1}{1+1} = 0.5$

continuing

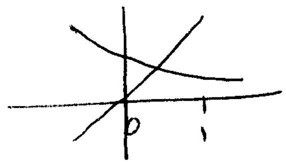
$$x_2 = 0.500 - \frac{-0.693 + 0.500}{2+1} = 0.564$$

$$x_3 = 0.564 - \frac{-0.573 + 0.564}{1.773 + 1} = 0.567$$

$$\boxed{0.567}$$

$$x_4 = 0.567 - \frac{-0.567 + 0.567}{1.764 + 1} = 0.567$$

#15 $f(x) = x$
$g(x) = e^{-x}$



$F = x - e^{-x}$
$F' = 1 + e^{-x}$

$$x_{n+1} = x_n - \frac{x_n - e^{-x_n}}{1 + e^{-x_n}}$$

let $x_0 = 0.5$ $\quad x_1 = 0.5 - \frac{-0.1065}{1.6065} = 0.5663$

$$x_2 = 0.5663 - \frac{-0.00132}{1.5672} = 0.5671$$

$$\boxed{0.5671}$$

$$x_3 = 0.5671 - \frac{-98 \times 10^{-5}}{1.5672} = 0.5671$$

(note: same as above since $\ln(x) = \ln(e^{-x}) = -x$ or $\underline{x + \ln x = 0}$

#39 $\sqrt[4]{6}$ $\quad f(x) = x^4 - 6 = 0$
$f'(x) = 4x^3$

$$x_{n+1} = x_n - \frac{x_n^4 - 6}{4x_n^3}$$

$x_0 = 1.5$

$$x_1 = 1.5 - \frac{-0.9375}{13.5} = 1.5694$$

$$\boxed{1.5651}$$

$$x_2 = 1.5694 - \frac{0.06645}{15.4618} = 1.5651$$

$$x_3 = 1.5651 - \frac{0.000236}{15.3351} = 1.5651$$