

STRATEGIES for Evaluating INTEGRALS - Prof. Richard B. Goldstein

Integral	Strategies	simplified form
$\int \frac{x^2 + \sqrt{x} + 1}{x} dx$	algebraic simplification	$\int x + x^{-1/2} + \frac{1}{x} dx$
$\int \sin(4x) \cos(3x) dx$	trigonometric identity	$\int \frac{1}{2} [\sin(4-3)x + \sin(4+3)x] dx$
$\int \frac{2x+5}{x-3} dx$	algebraic simplification substitution	$\int \left(2 + \frac{11}{x-3}\right) dx$, then let $u = x - 3$
$\int x^2 \sin(2x) dx$	integration by parts repeat twice	let $u = x^2$ let $dv = \sin(2x) dx$
$\int \sin^4 x \cos^5 x dx$	substitution	let $u = \sin(x)$, then $du = \cos(x) dx$ and $\cos^4 x = (1 - u^2)^2$
$\int \frac{2x^2 - x + 4}{x^3 + 4x} dx$	partial fractions	$\int \left(\frac{1}{x} + \frac{x}{x^2 + 4} - \frac{1}{x^2 + 4}\right) dx$
$\int \frac{x^3 - x^2}{(x-6)(5x+3)^3} dx$	partial fractions	$\int \left(\frac{A}{x-6} + \frac{B}{5x+3} + \frac{C}{(5x+3)^2} + \frac{D}{(5x+3)^3}\right) dx$
$\int \frac{1}{x - \sqrt{x+2}} dx$	substitution partial fractions	let $u = \sqrt{x+2} \rightarrow \int \frac{2u}{u^2 - u - 2} du$ $= \int \left(\frac{4/3}{u-2} + \frac{2/3}{u+1}\right) du$
$\int \frac{dx}{\sqrt{5-4x-x^2}}$	complete the square substitution	$\int \frac{dx}{\sqrt{9-(x+2)^2}}$, then let $u = x + 2$
$\int \frac{dx}{\sqrt{x} - \sqrt[3]{x}}$	rational substitution	let $u = \sqrt[6]{x}$ or $x = u^6 \rightarrow \int \frac{6u^5}{u^3 - u^2} du$ $= 6 \int \left(u^2 + u + 1 + \frac{1}{u-1}\right) du$