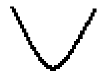
Graphing Functions - Prof. Richard B. Goldstein

Shapes

$f'(x) < 0$ decreasing  $f'(x) > 0$ increasing 

$f''(x) > 0$ concave up  $f''(x) < 0$ concave down 

Maximum and Minimum Points

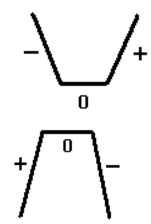
Absolute (Global) Maximum Point: $f(x) \le f(c)$ for all x in domain D
Absolute (Global) Minimum Point: $f(x) \ge f(c)$ for all x in domain D

Local (Relative) Maximum Point $f(x) \le f(c)$ for all x in some open interval containing c
Local (Relative) Minimum Point $f(x) \ge f(c)$ for all x in some open interval containing c
Note: at an endpoint c use a half-open interval in domain D

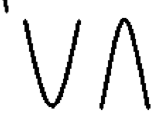
For example: (-1, 0) and (1, 0) are local minimum points for $\sqrt{1-x^2}$ which has a domain of [-1, 1] but not for $1 - x^2$ on [-1, 1] which is assumed to have a domain of all real numbers $(-\infty, \infty)$.

Critical Points: points at which $f'(c) = 0$ or $f'(c)$ D.N.E.

First Derivative Test: c is a critical point and for a
minimum: $f'(c^-) < 0$ and $f'(c^+) > 0$
maximum: $f'(c^-) > 0$ and $f'(c^+) < 0$



Second Derivative Test: c is a critical point for which $f'(c) = 0$ and
$f''(c) > 0$ for a minimum
$f''(c) < 0$ for a maximum.
If $f''(c) = 0$, then the test fails.

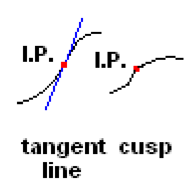


$f''(c) > 0$ $f''(c) < 0$

Absolute Maximums and Minimums: Compare endpoints and all critical points.

Inflection Points

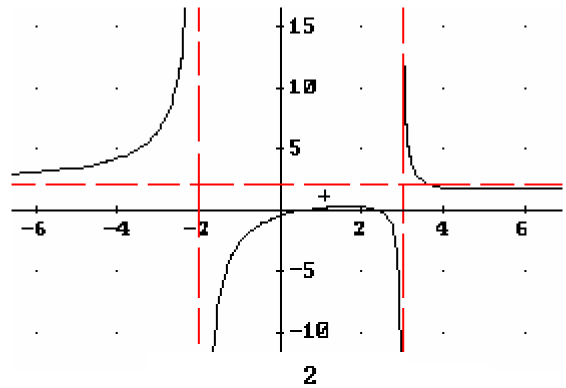
any point c where the function is continuous and $f''(c^-)$ and $f''(c^+)$ are of opposite sign; there is a tangent line at c (even if it is vertical) or $f'(c)$ D.N.E.



For example, $x^{1/3}$ has a vertical tangent at the inflection point (0, 0).

Also, $f(x) = \begin{cases} -x(x+1) \text{ for } x \le 0 \\ 4x(x-1) \text{ for } x > 0 \end{cases}$ has no tangent at the cusp (0, 0). However, the function is continuous at (0, 0) and is curved down for x < 0 and curved up for x > 0.

Asymptotes for $R_{m,n} = \dfrac{P_m(x)}{Q_n(x)} = \dfrac{a_m x^m + a_{m-1}x^{m-1} + \cdots + a_1 x + a_0}{b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0}$



Graph of $f(x) = \dfrac{2 \cdot x^2 - 6 \cdot x + 3}{x^2 - x - 6}$

<u>Vertical</u>: Consider points where $Q_n(x) = 0$ (roots of Q_n) and evaluate on both left (c^-) and right (c^+)

<u>Horizontal</u>: $\begin{cases} m > n \text{ then } \pm\infty \\ m = n \text{ then } \dfrac{a_m}{b_m} \\ m < n \text{ then } 0 \end{cases}$

Consider f(x) as x becomes large (forexample, over 100 or below -100)

L'Hospital's Rule

Suppose f and g are differentiable and $g'(x) \neq 0$ on open interval *I* that contains *a* (except possibly at *a*). Suppose that

$\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$

or that $\lim_{x \to a} f(x) = \pm\infty$ and $\lim_{x \to a} g(x) = \pm\infty$ (That is, indeterminate forms $\frac{0}{0}$ or $\frac{\infty}{\infty}$). Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

if the limit on the right side exists, or is ∞ or -∞.

Indeterminate Forms

0/0 or ∞/∞ will use L'Hospital's Rule

$0 \cdot \infty = f \cdot g$ is changed to $\dfrac{f}{1/g}$ or $\dfrac{g}{1/f}$

∞ - ∞ is changed algebraically to one fraction with a common denominator

0^0, ∞^0, 1^∞ can be found by taking natural logarithms

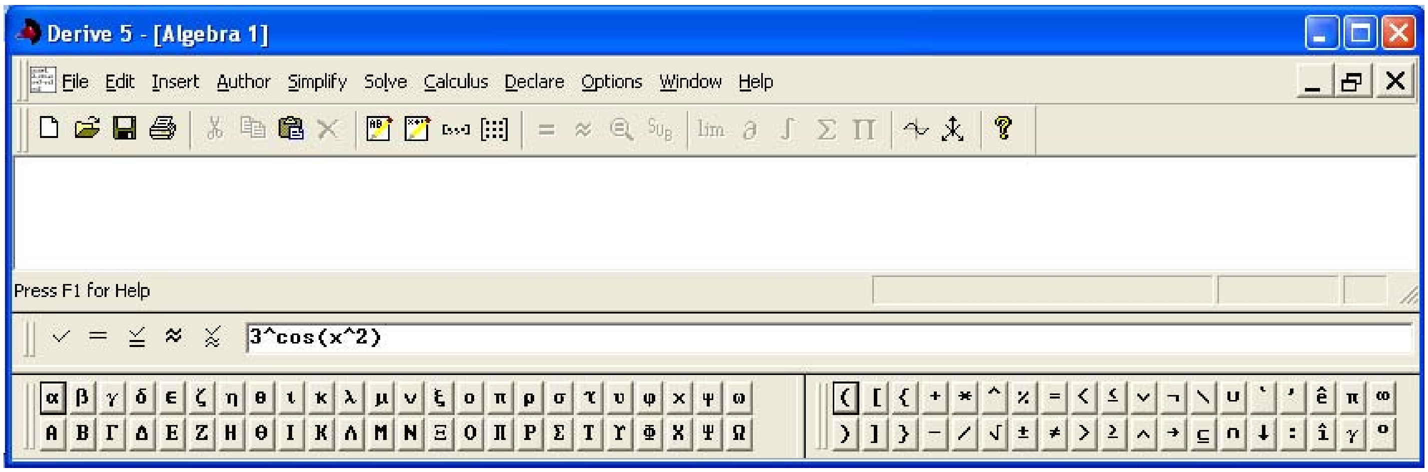
Let $y = [f(x)]^{g(x)} \Rightarrow \ln y = g(x) \ln f(x)$

Find $\lim_{x \to a} \ln y = \lim_{x \to a} g(x) \ln f(x)$, then find e to that power.

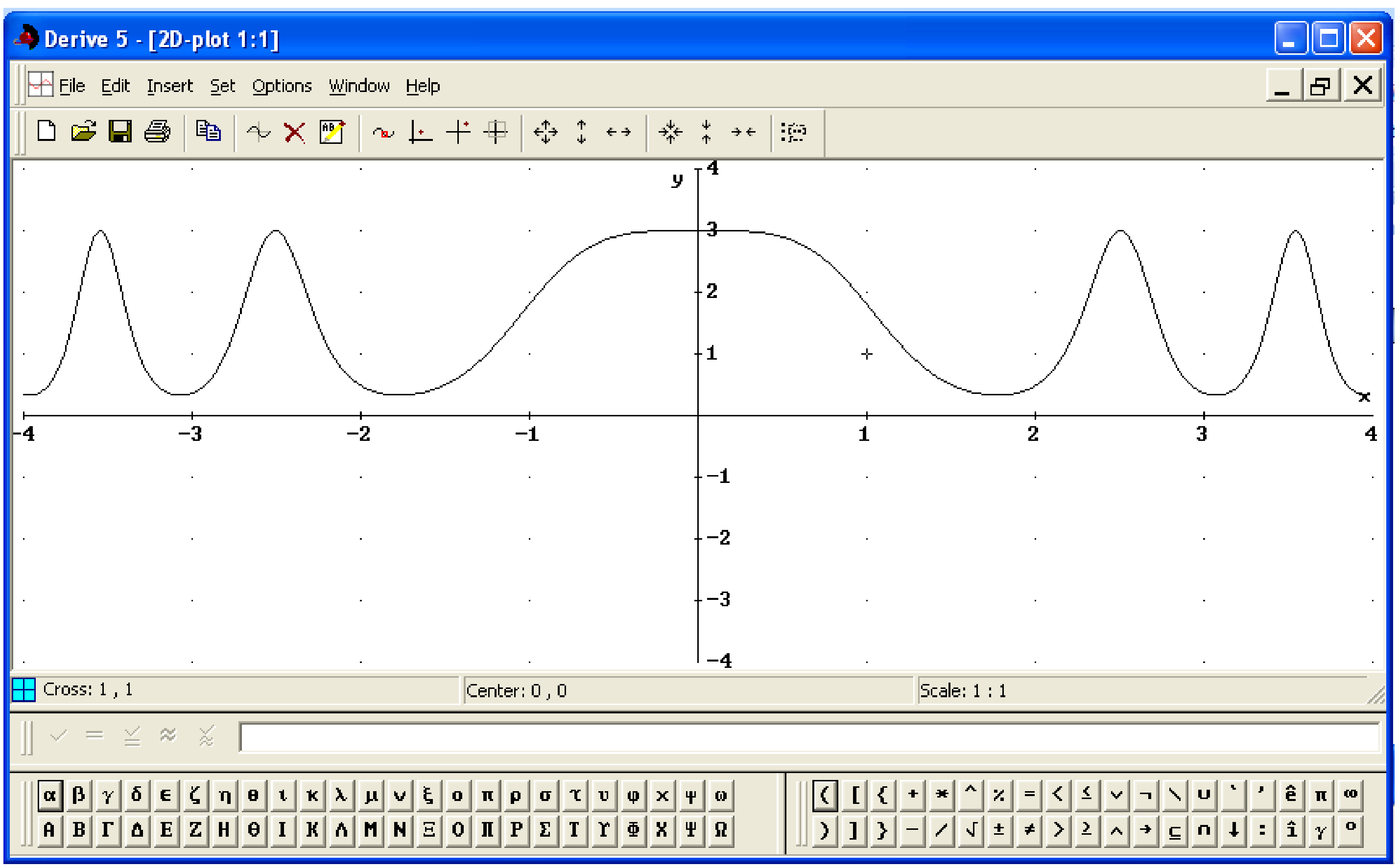
GRAPHING FUNCTIONS using DERIVE and MATHEMATICA

DERIVE

Type the equation in the user space and press the icon  twice



Use the icon  to return to the algebra window



For parametric equations: enter as [sin(3t), cos(7t+sin(3t))] and choose the range -3.14 to 3.14

MATHEMATICA

Untitled-1 *

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In[1]:= Plot[3^Cos[x^2], {x, -4, 4}]
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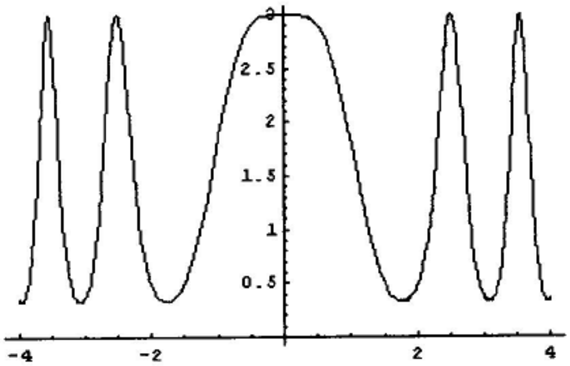


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Out[1]= - Graphics -
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In[4]:= ParametricPlot[{Sin[3 t], Cos[7 t + Sin[3 t]]}, {t, -Pi, Pi}]
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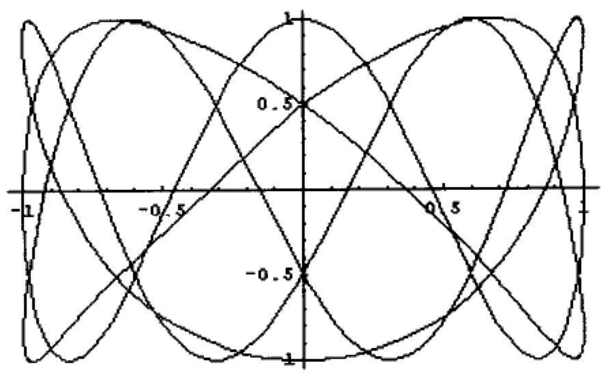


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Out[4]= - Graphics -
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- Plot [*f*, {*x*, *xmin*, *xmax*}] generates a plot of f as a function of *x* from *xmin* to *xmax*.
- Plot [{f_1, f_2, …}, {*x*, *xmin*, *xmax*}] plots several functions f_i
- ParametricPlot3D [{f_x, f_y, f_z}, {*t*, *tmin*, *tmax*}] produces a three-dimensional space curve parametrized by a variable t which runs from *tmin* to *tmax*