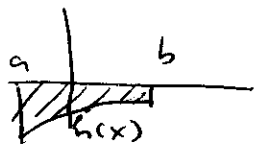
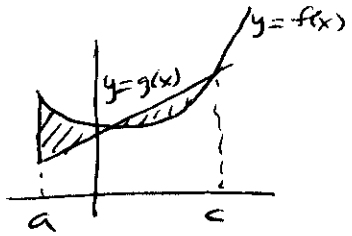
Math 108 – Prof. Richard B. Goldstein – Chap 7 HW – Barnett 10th

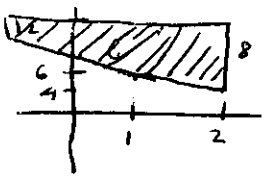
Sect 7.1

#3 $\int_a^b [0 - h(x)]\,dx$





#23 $\int_a^b [f(x) - g(x)]\,dx + \int_b^c [g(x) - f(x)]\,dx$

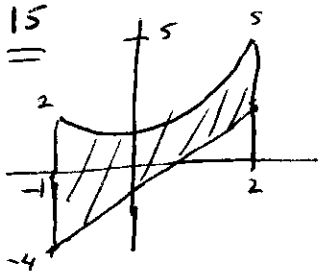


#33 $y = -2x + 8 \quad -1 \le x \le 2$

$y = 12$

Trapezoid $= 3\left(\frac{8+2}{2}\right) = 15$

$$\int_{-1}^{2} 12 - (-2x+8)\,dx = \int_{-1}^{2} 4 + 2x\,dx = 4x + x^2\Big|_{-1}^{2} = 12 - (-3) = 15$$



#34 $y = x^2 + 1 \quad y = 2x - 2 \quad -1 \le x \le 2$

$$\int_{-1}^{2} (x^2+1) - (2x-2)\,dx$$

$$= \int_{-1}^{2} x^2 - 2x + 3\,dx = \left(\frac{x^3}{3} - x^2 + 3x\right)\Big|_{-1}^{2} = \frac{14}{3} - \left(\frac{-13}{3}\right) = \frac{27}{3} = 9$$

#69 Gini Index $= 2\int_0^1 [x - f(x)]\,dx$

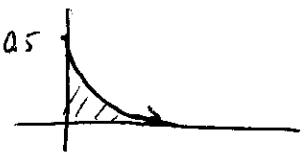
(A) $f(x) = x^{2.4}$ 1935 $\quad 2\int_0^1 x - x^{2.4}\,dx = 2\left[\frac{x^2}{2} - \frac{x^{3.4}}{3.4}\right]_0^1 = 0.412$

(B) $f(x) = x^{1.6}$ 1947 $\quad 2\int_0^1 x - x^{1.6}\,dx = 2\left[\frac{x^2}{2} - \frac{x^{2.6}}{2.6}\right]_0^1 = 0.231$

income was more evenly distributed in 1947

Sect 7.2

#7 $f(x) = \begin{cases} \frac{2}{(x+2)^2} & \text{if } x \ge 0 \\ 0 & \text{otherwise} \end{cases}$



(A) $\int_0^6 \frac{2}{(x+2)^2}\,dx = \int_0^6 2(x+2)^{-2}\,dx = \frac{2(x+2)^{-1}}{-1}\Big|_0^6 = \left(-\frac{2}{8}\right) - (-1) = \frac{3}{4} = 0.75$

(B) $\int_6^{12} \frac{2}{(x+2)^2}\,dx = -\frac{2}{(x+2)}\Big|_6^{12} = -\frac{2}{14} - \left(-\frac{2}{8}\right) = \frac{2}{8} - \frac{2}{14} = 0.107$

#19 $f(t) = 400\,e^{0.05t}$

$$I = \int_0^3 400\,e^{0.05t}\,dt = \frac{400\,e^{0.05t}}{0.05}\Big|_0^3 = 8000\left(e^{.15} - 1\right) = \$1,294.67$$

Sect 7.2 Continued

#29 $FV = e^{rT}\int_0^T f(t)\,e^{-rt}\,dt$ $\quad r = 10\% = 0.1$, $T = 5$

Clothing $e^{0.5}\int_0^5 12000e^{-0.1t}\,dt = e^{0.5}\left(\frac{12000e^{-0.1t}}{-0.1}\right)\Big|_0^5 = \$77{,}847$

Computer $e^{0.5}\int_0^5 10000^{0.05t}e^{-0.1t}\,dt = e^{0.5}\int_0^5 10000e^{-0.05t}\,dt$

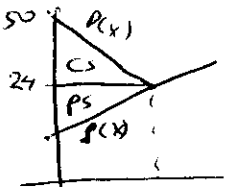
$= e^{0.5}\left(\frac{10000e^{-0.05t}}{-0.05}\right)\Big|_0^5 = \$72{,}939$

∴ Clothing is better in the first 5 yrs

#45 $p = D(x) = 50 - 0.1x$ $\quad p = S(x) = 11 + 0.05x$

$50 - 0.1x = 11 + 0.05x$

$39 = 0.15x \Rightarrow \bar{x} = 39/0.15 = 260$ $\quad p = 50 - 26 = 24$



$CS = \int_0^{260} 50 - 0.1x - 24\,dx = 26x - 0.05x^2\Big|_0^{260} = \$3380$

$PS = \int_0^{260} 24 - (11 + 0.05x)\,dx = 13x - 0.025x^2\Big|_0^{260} = \$1690$

Sect 7.3

#7 $\int xe^{-x}\,dx$ $\quad u = x$, $du = dx$; $dv = e^{-x}dx$, $v = -e^{-x}$

$-xe^{-x} - \int(-e^{-x})\,dx = -xe^{-x} - e^{-x} + c$

or $-(x+1)e^{-x} + c$

#17 $\int \frac{\ln x}{x}\,dx$ $\quad u = \ln x$, $du = \frac{1}{x}$; $dv = \frac{1}{x}dx$, $v = \ln x$

$(\ln x)^2 - \int \frac{\ln x}{x}\,dx$

$\int \frac{\ln x}{x}\,dx = (\ln x)^2 - \int \frac{\ln x}{x}\,dx \Rightarrow \int \frac{\ln x}{x}\,dx = \frac{(\ln x)^2}{2}$

Can also be done by subst. let $u = \ln x$, $du = \frac{1}{x}dx$ $\quad \int u\,du = \frac{u^2}{2}$ ↗

#47 $\int_0^5 2t - te^{-t}\,dt = t^2\Big|_0^5 - \int_0^5 te^{-t}\,dt = 25 + (t+1)e^{-t}\Big|_0^5 = 25 + (0.0404 - 1) = 24.04$

↑ from #7 above

#64 $R(t) = te^{-0.2t}$

$\int_0^{10} te^{-0.2t}\,dt$ $\quad u = t$, $du = dt$; $dv = e^{-0.2t}dt$, $v = \frac{e^{-0.2t}}{-0.2}$

$\frac{te^{-0.2t}}{-0.2}\Big|_0^{10} - \int_0^{10}\frac{e^{-0.2t}}{-0.2}\,dt$

$= -6.767 + \frac{e^{-0.2t}}{-0.04}\Big|_0^{10} = -6.767 + (-3.383 + 25) = 14.85$