Math 108 - Prof. Richard B. Goldstein - Chap 6 HW - Barnett 10th

Sect 6.1 #7 $\int 5t^{-3}\,dt = \dfrac{5t^{-2}}{-2} + c = -\dfrac{5}{2t^2} + c$

#19 $\int 7t^{-4/3}\,dt = 7\dfrac{t^{-1/3}}{-1/3} + c = -21t^{-1/3} + c$ or $-\dfrac{21}{\sqrt[3]{t}} + c$

#49 $\int (5e^z + 4)\,dz = 5e^z + 4z + C$ $\quad \dfrac{d}{dz}(5e^z + 4z + c) = 5e^z + 4$ ✓

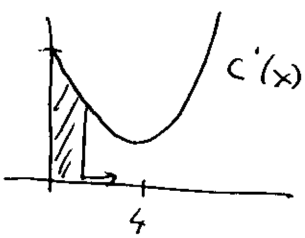
#71 $\dfrac{dy}{dx} = 2x^{-2} + 3x^{-1} - 1 \quad y(1) = 0$

$y = \dfrac{2x^{-1}}{-1} + 3\ln|x| - x + c \qquad y = -\dfrac{2}{x} + 3\ln|x| - x + c$

$y(1) = -2 + 0 - 1 + C = 0 \Rightarrow c = 3$

$\therefore\ y = -\dfrac{2}{x} + 3\ln|x| - x + 3$

#101



(A) The cost function will increase from 0 to 8. There is an inflection point at $x = 4$, where the curve is CD from 0 to 4 and CU from 4 to 8.

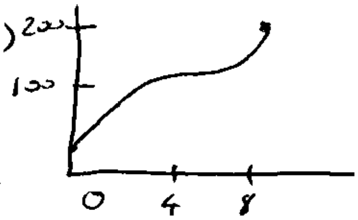
(B) $C'(x) = 3x^2 - 24x + 53$

$C(x) = \int C'(x)\,dx = x^3 - 12x^2 + 53x + K \qquad K = 30$ since fixed cost

$C(4) = 64 - 192 + 212 + 30 = 114 \qquad \$114{,}000$

$C(8) = 512 - 768 + 424 + 30 = 198 \qquad \$198{,}000$

(C)



(D) Text - manufacturing plants are often inefficient at low and high levels

Sect 6.2

#3 $\int (x^2-1)^5(2x)\,dx \quad u = x^2 - 1,\ \dfrac{du}{dx} = 2x \Longrightarrow \int u^5\,du = \dfrac{u^6}{6} + C = \dfrac{(x^2-1)^6}{6} + c$

#17 $\int (t^2+1)^5\,t\,dt \quad u = t^2 + 1,\ \dfrac{du}{dx} = 2t\,dt,\ \dfrac{du}{2} = t\,dt \Longrightarrow \int u^5 \dfrac{du}{2} = \dfrac{u^6}{12} + c = \dfrac{(t^2+1)^6}{12} + c$

#19 $\int x e^{x^2}\,dx \quad u = x^2,\ du = 2x\,dx,\ \dfrac{du}{2} = x\,dx \Longrightarrow \int e^u \dfrac{du}{2} = \dfrac{e^u}{2} + C = \dfrac{e^{x^2}}{2} + c$

#48 $\int x^2(2x^3+1)^{1/2}\,dx \quad u = 2x^3 + 1,\ du = 6x^2\,dx,\ \dfrac{du}{6} = x^2\,dx \Longrightarrow \int u^{1/2}\dfrac{du}{6} = \dfrac{u^{3/2}}{6(3/2)} + C = \dfrac{u^{3/2}}{9} + C = \dfrac{(2x^3+1)^{3/2}}{9} + c$

Sect 6.2 continued

Math 108 (2)
Prof. R.B. Goldstein
Chap 6 HW
Barnett - 10th

#71 $S'(t) = 10 - 10e^{-0.1t}$ $\quad 0 \le t \le 24$ $\quad$ note: $\int e^{kt}\,dt = \frac{e^{kt}}{k}$

(A) $S(t) = \int 10 - 10e^{-0.1t} = 10t - 10\frac{e^{-0.1t}}{(-0.1)} + k = \underline{10t + 100e^{-0.1t} - 100}$ since $S(0) = 0$

(B) $S(12) = 10(12) + 100e^{-1.2} - 100 = 120 + 30.12 - 100 = \underline{\$50.12}$ million

(c) $S(t) = \underline{10t + 100e^{-0.1t} - 100} = 100$ graphically solve for $t \approx \underline{18.4}$ mos

Sect 6.3

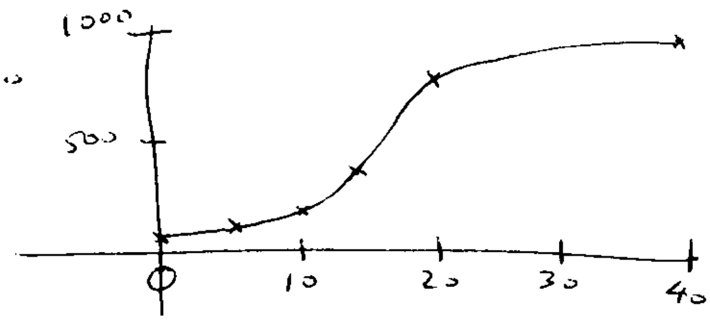
#3 $\frac{dy}{dx} = \frac{7}{x}$ $\quad y = \int \frac{7}{x}\,dx = 7\ln|x| + c$ $\quad \underline{y = 7\ln|x| + c}$

#7 $\frac{dy}{dx} = x^2 - x$; $y(0) = 0$ $\quad y = \int x^2 - x\,dx = \frac{x^3}{3} - \frac{x^2}{2} + c$ $\quad y(0) = 0 \Rightarrow c = 0$

$$y = \frac{x^3}{3} - \frac{x^2}{2}$$

#41 $N = \dfrac{1000}{1 + 999e^{-0.4t}}$ $\quad 0 \le t \le 40$, $\quad 0 \le N \le 1000$

t	N
0	1
5	7.3
10	51.8
20	749
40	999.9
15	287.7



#53 (A) $\frac{dN}{dt} = k(L - N)$ $\quad N(0) = 0$, $N(10) = 0.4L$ $\quad$ Limited Growth Model $\quad N(t) = L(1 - e^{-kt})$

$N(10) = L(1 - e^{-10k}) = 0.4L$

$1 - e^{-10k} = 0.4 \Rightarrow$ $\quad e^{-10k} = 0.6$

$-10k = \ln(0.6) = -0.5108 \Rightarrow \underline{k = 0.05108}$

$\therefore \underline{N(t) = L(1 - e^{-0.05108t})}$

(B) $N(5) = L(1 - e^{-0.05108(5)}) = \underline{0.225L}$

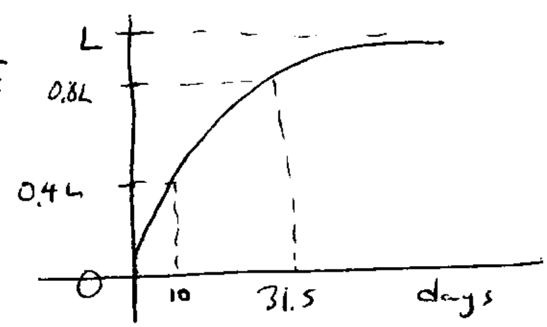
(c) $L(1 - e^{-0.05108t}) = 0.8L$

$1 - e^{-0.05108t} = 0.8$

$e^{-0.05108t} = 0.2$

$-0.05108t = \ln(0.2) = -1.6094$

$\underline{t = 31.5 \text{ days}}$



Math 108 (3)
Prof. R.B. Goldstein
Chap 6 HW
Barnett - 10th

Sect 6.3 Continued

#55 $\frac{dI}{dx} = -kI \quad I(0) = I_0$ Exponential Decay Model $I = Ce^{-kx}$

$I(0) = C = I_0 \quad \therefore \quad \underline{\underline{I(x) = I_0 e^{-kx}}}$

$k = 0.00942 \qquad \underline{\underline{I(x) = I_0 e^{-0.00942x}}}$

$I_0 e^{-0.00942x} = 0.5 I_0$

$e^{-0.00942x} = 0.5$

$-0.00942x = \ln(0.5) = -0.69315 \Rightarrow x = \frac{-0.69315}{-0.00942} = \underline{\underline{73.6 \text{ ft}}}$

Sect 6.4

#3

x	$f(x)$ (A)	$g(x)$ (B)
1	1	8
2	5	4
3	7	2
4	8	1

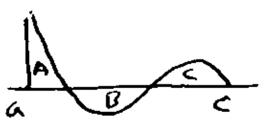
(A) $L_3 = 1(1+5+7) = 13$
$R_3 = 1(5+7+8) = 20$

(B) $L_3 = 1(8+4+2) = 14$
$R_3 = 1(4+2+1) = 7$

#9 $f(x) = 25 - 3x^2 \quad [-2, 8] \quad h = \frac{8-(-2)}{5} = 2$ midpts: $-1, +1, +3, +5, +7$

$M_5 = 2[f(-1) + f(1) + f(3) + f(5) + f(7)] = 2[22 + 22 + (-2) + (-50) + (-122)] = \underline{\underline{-260}}$

#19 $\int_a^c f(x)dx$



Area A = 1.408
B = 2.475
C = 5.333

$1.408 - 2.475 + 5.333 = \underline{\underline{4.266}}$

#43 $L_{10} = 100(0 + 183 + 235 + 245 + 260 + 286 + 322 + 388 + 453 + 489) = 286{,}100$

$|f(1000) - f(0)| \frac{1000-0}{n} = |500 - 0| \frac{(1000-0)}{n} = \frac{500000}{n} \le 2500$

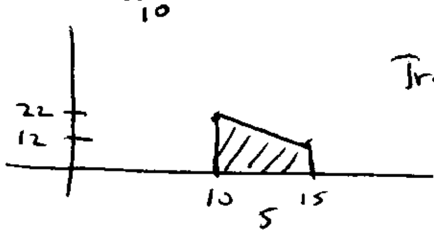
$\Rightarrow n \geq \frac{500000}{2{,}500} = \underline{\underline{200}}$

Sect 6.5

#3 $F(x) = -x^2 + 42x + 240$

(A) $F(15) - F(10) = \left[-x^2 + 42x + 240\right]_{10}^{15} = (645 - 560) = \underline{\underline{85}}$

(B) $F'(x) = -2x + 42$



Trapezoid
$A = \frac{1}{2}(22 + 12)5 = \frac{1}{2}(34)5 = \underline{\underline{85}}$

Sect 6.5 Continued

Math 108 (4)
Prof. R.B. Goldstein
Chap 6 HW
Barnett -10th

#11 $\int_{-3}^{4}(4-x^2)\,dx = 4x - \frac{x^3}{3}\Big|_{-3}^{4} = (-5.3\overline{3}) - (-3) = \underline{\underline{-2.3\overline{3}}}$

#15 $\int_{0}^{1} e^{2x}\,dx = \frac{e^{2x}}{2}\Big|_{0}^{1} = \frac{e^2 - e^0}{2} = \frac{e^2-1}{2} \simeq \underline{\underline{3.1945}}$

#17 $\int_{1}^{3.5} 2x^{-1}\,dx = 2\ln|x|\Big|_{1}^{3.5} = 2\ln 3.5 - 2\underbrace{\ln 1}_{0} = 2\ln 3.5 \approx \underline{\underline{2.5055}}$

#73 $p = S(x) = 10(e^{0.02x} - 1)$ $[20,30]$ find average

$\frac{1}{30-20}\int_{20}^{30} 10\,(e^{0.02x}-1)\,dx = \frac{1}{10}\,10\left(\frac{e^{0.02x}}{0.02} - x\right)\Big|_{20}^{30}$

$= \left(\frac{e^{0.6}}{0.02} - 30\right) - \left(\frac{e^{0.4}}{0.02} - 20\right) = 61.106 - 54.591 = \underline{\underline{6.515}}$