Math 108 – Prof. Richard B. Goldstein – Chap 4 HW – Barnett 2nd Ed.

Sect 4.1

#7 $3 = e^{0.1r} \Rightarrow 0.1r = \ln 3$ $\quad r = \frac{\ln 3}{0.1} = \frac{1.0986}{0.1} = \boxed{10.986}$

#19 $A = Pe^{rt}$ $\quad 20000 = Pe^{0.052(10)} = 1.682P$ $\quad P = \frac{20000}{1.682\ldots} = \boxed{\$11,890.41}$

#25 $A = Pe^{rt}$ $\quad A = 2P = Pe^{0.07t} \Rightarrow e^{0.07t} = 2$ $\quad 0.07t = \ln 2$

$t = \frac{\ln 2}{0.07} = \frac{0.693\ldots}{0.07} = \boxed{9.9 \text{ yrs}}$ (note: by rule of 70: $\frac{70}{7} = 10$)

#29 $20000 \rightarrow 10000 \Rightarrow 10000e^{0.072t}$
$\searrow 10000 \Rightarrow 10000(1+0.084)^t$ } $10000e^{0.07t} + 10000(1.084)^t = 35{,}000$

by trial & error or graphics $t \approx 7.3$ years

for example $10000e^{0.072(7.3)} + 10000(1.084)^{7.3} = 16{,}915 + 18{,}018 = 34{,}933$ ← closer

and $10000e^{0.072(7.4)} + 10000(1.084)^{7.4} = 17{,}037 + 18{,}164 = 35{,}201$

Sect 4.2

#1 $f(x) = 5e^x + 3x + 1$ $\quad f'(x) = \underline{5e^x + 3}$

#7 $f(x) = e^x + x - \ln x$ $\quad f'(x) = e^x + 1 - \frac{1}{x}$

#19 $f(x) = \ln x^3$; $x = e$ $\quad f(e) = \ln e^3 = 3 \ln e = 3(1) = 3$

$f'(x) = \frac{1}{x^3} \cdot 3x^2 = \frac{3}{x}$ or $f(x) = 3\ln x$, $f'(x) = \frac{3}{x}$ $\quad f'(e) = \frac{3}{e}$

$y - 3 = \frac{3}{e}(x - e) = \frac{3x}{e} - 3$ $\quad \boxed{y = \frac{3x}{e}}$

#31 $y = \log_2 x$ $\quad y' = \frac{1}{\ln 2}\left(\frac{1}{x}\right)$ (p228)

#37 $y = 10 + x + 10^x$ $\quad y' = 1 + 10^x(\ln 10)$ (p228)

$\boxed{\frac{d}{dx}\log_b x = \frac{1}{\ln b}\left(\frac{1}{x}\right)}$

$\boxed{\frac{d}{dx}b^x = b^x \ln b}$

Math 108 (2)
Prof. R. B. Goldstein
Chap 4 HW
Barnett - 2nd Ed.

Sect 4.3

#1 $f(x) = 2x^3(x^2-2)$ $\quad f'(x) = 6x^2(x^2-2) + 2x^3(2x)$
$= 10x^4 - 12x^2$

#7 $f(x) = \dfrac{2x+3}{x-2}$ $\quad f'(x) = \dfrac{(x-2)(2) - (2x+3)(1)}{(x-2)^2} = \boxed{\dfrac{-7}{(x-2)^2}}$

#23 $f(x) = \dfrac{e^x}{x^2+1}$ $\quad f'(x) = \dfrac{(x^2+1)e^x - e^x(2x)}{(x^2+1)^2} = \boxed{\dfrac{e^x(x^2-2x+1)}{(x^2+1)^2} = \dfrac{(x-1)^2 e^x}{(x^2+1)^2}}$

#43 $y = \dfrac{5x-3}{x^2+2x}$ $\quad f'(x) = \dfrac{(x^2+2x)(5) - (5x-3)(2x+2)}{(x^2+2)^2} = \boxed{\dfrac{-5x^2+6x+6}{(x^2+2x)^2}}$

#83 $S'(t) = \dfrac{90t^2}{t^2+50}$

(A) $S'(t) = \dfrac{(t^2+50)(180t) - (90t^2)(2t)}{(t^2+50)^2} = \boxed{\dfrac{9000t}{(t^2+50)^2}}$

(B) $S(10) = \dfrac{9000}{150} = \boxed{60}$ $\quad S'(10) = \dfrac{90000}{22500} = \boxed{4}$

After 10 months, the total sales were 60,000 and increasing at the rate 4,000 per month

(C) $S(11) \approx S(10) + (11-10)S'(10) = 60 + 1 \cdot 4 = \boxed{64{,}000}$ cds

Sect 4.4

#5 $y = (3x^2 - x + 5)^4$ $\quad y = f(u) = u^4$ $\quad u = g(x) = 3x^2 - x + 5$

#23 $f(x) = (3x^2+5)^5$ $\quad f'(x) = 5(3x^2+5)^4 \cdot (6x) = 30x(3x^2+5)^4$

#41 $f(x) = 5e^{x^2-4x+1}$; $x = 0$ $\quad f(0) = 5e^1 = 5e$

$f'(x) = 5e^{x^2-4x+1} \cdot (2x-4)$ $\quad f'(0) = 5(e^1)(-4) = -20e$

$y - 5e = -20e(x-0)$ $\quad \boxed{y = -20xe + 5e}$

horizontal when $f'(x) = 0$ $\quad 2x - 4 = 0$ at $\boxed{x = 2}$

#49 $g(x) = 4xe^{3x}$ $\quad g'(x) = 4 \cdot e^{3x} + 4xe^{3x} \cdot 3 = \boxed{(4+12x)e^{3x}}$

#65 $f(x) = \dfrac{x}{(2x-5)^3}$; $x = 3$ $\quad f(3) = \dfrac{3}{1^3} = 3$

$f'(x) = \dfrac{(2x-5)^3 \cdot 1 - x3(2x-5)^2 \cdot 2}{(2x-5)^6} = \dfrac{(2x-5) - 6x}{(2x-5)^4} = \dfrac{-4x-5}{(2x-5)^4}$

$f'(3) = \dfrac{-17}{1^4} = -17$

$y - 3 = -17(x-3)$

$\boxed{y = -17x + 54}$

Math 108 (3)
Prof. R.B. Goldstein
Chap 4 HW
Barnett - 2nd Ed.

Sect 4.5

#1 $3x + 5y + 9 = 0$

(A) $3 + 5y' + 0 = 0$

$\boxed{y' = -\frac{3}{5}}$

(B) $5y = -3x - 9$

$y = -\frac{3}{5}x - \frac{9}{5}$ $\quad \boxed{y' = -\frac{3}{5}}$

#9 $y^2 + 2y + 3x = 0$; $(-1, 1)$ $\quad 1^2 + 2(1) + 3(-1) = 0$ ✓

$2yy' + 2y' + 3 = 0$

$2(1)y' + 2y' + 3 = 0$ $\quad 4y' = -3$ $\quad \boxed{y' = -\frac{3}{4}}$

#15 $x^2y - 3x^2 - 4 = 0$; $(2, 4)$ $\quad 4(4) - 3(2)^2 - 4 = 16 - 12 - 4 = 0$ ✓

$2xy + x^2y' - 6x = 0$ $\quad 2(2)(4) + (2)^2y' - 6(2) = 0$ $\quad 4y' = -4$

$16 + 4y' - 12 = 0$ $\quad \boxed{y' = -1}$

#19 $x^3 - y = \ln y$; $(1, 1)$ $\quad 1^3 - 1 = 0 = \ln 1$ ✓

$3x^2 - y' = \frac{1}{y}y'$ $\quad 3(1)^2 - y' = (1)y'$ $\quad 2y' = 3$ $\quad \boxed{y' = 3/2}$

Sect 4.6

#3 $x^2 + y^2 = 1$ $\quad \frac{dy}{dt} = -4$ $\quad x = -0.6$ $\quad y = 0.8$ $\quad$ find $\frac{dx}{dt}$

$2x\dot{x} + 2y\dot{y} = 0$ $\quad 2(-0.6)\dot{x} + 2(0.8)(-4) = 0$

$-1.2\dot{x} = 6.4$ $\quad \dot{x} = \frac{6.4}{-1.2} = \boxed{-\frac{16}{3}}$

#9

$y = \sqrt{x^2 - 16}$

$y^2 + 4^2 = x^2$

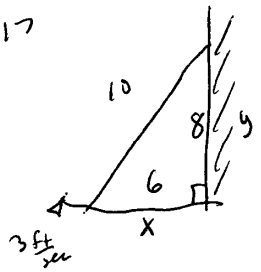
$2y\dot{y} = 2x\dot{x}$

$\dot{x} = -3$ $\quad y = 30$ $\quad x = \sqrt{4^2 + 30^2} = 30.265$

$2(30)\dot{y} = 2(30.265)(-3)$

$\boxed{\dot{y} = -3.0265 \text{ ft/sec}}$

#17



$x^2 + y^2 = 10^2 = 100$

$2x\dot{x} + 2y\dot{y} = 0$

$2(6)(3) + 2(8)\dot{y} = 0$

$36 + 16\dot{y} = 0$

$\boxed{\dot{y} = -\frac{36}{12} = -2.25 \text{ ft/sec}}$

#27 $S = 60{,}000 - 40{,}000e^{-0.0005x}$

$x = 2000 + 300t$

$\frac{dS}{dt} = \frac{dS}{dx}\frac{dx}{dt} = (20e^{-0.0005x})(300) = 6000e^{-0.0005x}\Big|_{x=2000}$

$= 6000e^{-1} = \boxed{2{,}207.28/\text{wk}}$

Prof. R.B. Goldstein
Chap 4 HW
Barnett – 2nd Ed

Sect 4.7

#15 $x = f(p) = 950 - 2p - 0.1p^2$

$f'(p) = -2 - 0.2p$

$$E(p) = \frac{-p(-2-0.2p)}{950-2p-0.1p^2} = \frac{2p+0.2p^2}{950-2p-0.1p^2}$$

(A) $E(50) = \frac{600}{600} = 1$ Unit Elasticity

(B) $E(70) = \frac{1120}{320} = 3.5$

(C) $E(100) = \frac{2200}{-250} = -8.8$

} Elastic

(text answers are incorrect)

#17 (A) $p + 0.005x = 30 \Rightarrow 0.005x = 30 - p \Rightarrow \boxed{x = 6000 - 200p}$

$f'(p) = -200$

(B) $E(p) = \frac{-p(-200)}{6000-200p} = \boxed{\frac{200p}{6000-200p}}$

(C) $p = \$10$ $E(p) = \frac{2000}{4000} = \boxed{0.5}$ inelastic

$x(10) = 6000 - 200(10) = 4000$

$x(11) = 6000 - 200(11) = 3800$

} decr. of 200 or $\frac{200}{4000} = 5\%$

$10 \to 11$ is incr. of 10%

note: $\frac{5\%}{10\%} = 0.5$

(D) $p = \$25$ $E(25) = \frac{5000}{1000} = \boxed{5}$ elastic

$x(25) = 6000 - 200(25) = 1000$

$x(27.5) = 6000 - 200(27.5) = 500$

} decr. of 500 or $\frac{500}{1000} = 50\%$

$25 \to 27.5$ is incr. of 10%

note: $\frac{50\%}{10\%} = 5$

(E) $p = \$15$ $E(15) = \frac{3000}{3000} = \boxed{1}$ unit

$x(15) = 6000 - 200(15) = 3000$

$x(16.5) = 6000 - 200(16.5) = 2700$

} drop of 300 or $\frac{300}{3000} = 10\%$

$15 \to 16.5$ is incr. of 10%

note: $\frac{10\%}{10\%} = 1$