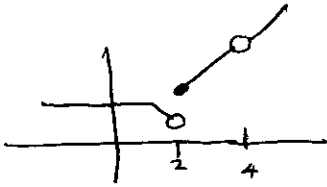


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Sect 3.1

#3



(A) $\lim_{x \rightarrow 2^-} f(x) = 1$

(D) $f(2) = 2$

(B) $\lim_{x \rightarrow 2^+} f(x) = 2$

(E) It is not possible to redefine $f(2)$ so that

(C) $\lim_{x \rightarrow 2} f(x)$ D.N.E.

$\lim_{x \rightarrow 2} f(x) = f(2)$ because of the gap

#27 $\lim_{x \rightarrow 1} \frac{2-f(x)}{x+g(x)} = \frac{2-(-5)}{1+4} = \frac{7}{5} = 1.4$ since denom $\neq 0$

#41 $f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ 2x & \text{if } x > 1 \end{cases}$

(A) $\lim_{x \rightarrow 1^+} f(x) = 2$

(C) $\lim_{x \rightarrow 1} f(x)$ D.N.E.

(B) $\lim_{x \rightarrow 1^-} f(x) = 1$

(D) $f(1)$ is not defined

#51 $f(x) = \frac{(x+2)^2}{x^2-4} = \frac{(x+2)^2}{(x+2)(x-2)}$

(A) $\lim_{x \rightarrow -2} f(x) \Rightarrow \lim_{x \rightarrow -2} \frac{x+2}{x-2} = \frac{0}{-4} = 0$ exists

(B) $\lim_{x \rightarrow 0} f(x) = \frac{4}{-4} = -1$

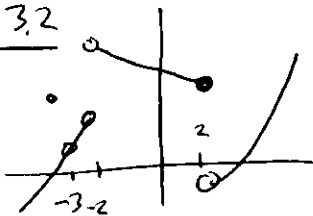
(C) $\lim_{x \rightarrow 2} f(x) \Rightarrow$ D.N.E. since denom. only goes to 0

#55 $f(x) = 3x+1$

$\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \rightarrow 0} \frac{[3(2+h)+1] - [3(2)+1]}{h} = \lim_{h \rightarrow 0} \frac{7+3h-7}{h} = \lim_{h \rightarrow 0} \frac{3h}{h} = 3$

Sect 3.2

#11



(A) $\lim_{x \rightarrow 3^-} g(x) = +1$

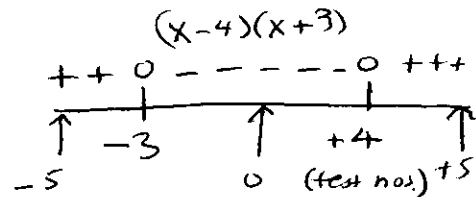
(B) $\lim_{x \rightarrow -3^+} g(x) = +1$

(C) $\lim_{x \rightarrow -3} g(x) = +1$

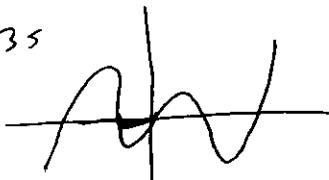
(D) $g(-3) = +3$

(E) No

#27 $x^2 - x - 12 < 0 \Rightarrow (x-4)(x+3) < 0$
for $(-3, 4)$



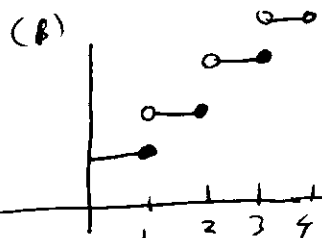
#35



(A) $f(x) > 0$ for $(-4, -2) \cup (0, 2) \cup (4, \infty)$

(B) $f(x) < 0$ for $(-\infty, -4) \cup (-2, 0) \cup (2, 4)$

#72 $R(x) = \begin{cases} 0.07 & \text{if } x \leq 1 \\ 0.12 & \text{if } 1 < x \leq 2 \\ 0.17 & \text{if } 2 < x \leq 3 \\ \text{etc.} \end{cases}$



(C) $R(x)$ is continuous at $x=3.5$

but not at $x=3$

sect 3.3

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#3 $f(x) = 3x^2$

(A) $\frac{f(4) - f(1)}{4 - 1} = \frac{48 - 3}{4 - 1} = \frac{45}{3} = 15$

(B) same as (A)

(C) $\frac{f(1+h) - f(1)}{1+h - 1} = \frac{3(1+h)^2 - 3}{h} = \frac{3(1+2h+h^2) - 3}{h} = \frac{3+6h+3h^2-3}{h} = \underline{\underline{6+3h}}$

(D) $\lim_{h \rightarrow 0} 6+3h = \underline{\underline{6}}$

(E) same as (D) $f(1) = 3$

(F) $y - 3 = 6(x - 1)$ $y - 3 = 6x - 6$ $\underline{\underline{y = 6x - 3}}$

#15 $f(x) = -x^2 + 4x - 9$

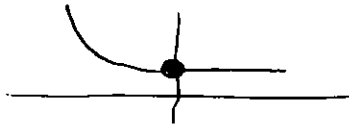
$\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{[-(1+h)^2 + 4(1+h) - 9] - [-1 + 4 - 9]}{h} = \lim_{h \rightarrow 0} \frac{(-h^2 + 2h - 6) - (-6)}{h} =$

$\lim_{h \rightarrow 0} \frac{-h^2 + 2h}{h} = \lim_{h \rightarrow 0} -h + 2 = \underline{\underline{2}}$

similar for $x=2, 3$
 $f'(2) = 0, f'(3) = -2$

#21 $\lim_{h \rightarrow 0} \frac{[5 + 3\sqrt{1+h}] - [5 + 3\sqrt{1}]}{h} = \lim_{h \rightarrow 0} \frac{3\sqrt{1+h} - 3}{h} = \lim_{h \rightarrow 0} \frac{3\sqrt{1+h} - 3}{h} \cdot \frac{3\sqrt{1+h} + 3}{3\sqrt{1+h} + 3}$
 $= \lim_{h \rightarrow 0} \frac{9(1+h) - 9}{h(3\sqrt{1+h} + 3)} = \lim_{h \rightarrow 0} \frac{9h}{3h(\sqrt{1+h} + 1)} = \lim_{h \rightarrow 0} \frac{3}{\sqrt{1+h} + 1} = \underline{\underline{\frac{3}{2}}}$

#49 $f(x) = \begin{cases} x^2 + 1 & x < 0 \\ 1 & x \geq 0 \end{cases}$



\mathbb{R}

on left $\lim_{h \rightarrow 0} \frac{(-h)^2 + 1 - 1}{-h} = \lim_{h \rightarrow 0} (-h) = 0$

on right $\lim_{h \rightarrow 0} \frac{1 - 1}{h} = 0$ \therefore diff everywhere

#61 $S'(t) = 2\sqrt{t+10}$

(A) $\lim_{h \rightarrow 0} \frac{2\sqrt{t+h+10} - 2\sqrt{t+10}}{h} = \lim_{h \rightarrow 0} \frac{2(\sqrt{t+h+10} - \sqrt{t+10})}{h} \cdot \frac{\sqrt{t+h+10} + \sqrt{t+10}}{\sqrt{t+h+10} + \sqrt{t+10}}$
 $= \lim_{h \rightarrow 0} \frac{2(t+h+10 - t-10)}{h(\sqrt{t+h+10} + \sqrt{t+10})} = \lim_{h \rightarrow 0} \frac{2h}{h(\sqrt{t+h+10} + \sqrt{t+10})} = \lim_{h \rightarrow 0} \frac{2}{\sqrt{t+h+10} + \sqrt{t+10}}$
 $= \frac{2}{2\sqrt{t+10}} = \boxed{\frac{1}{\sqrt{t+10}}}$

(B) $S(15) = 2\sqrt{25} = 10$
 $S'(15) = \frac{1}{\sqrt{25}} = 0.2$

(C) $S(11) \approx 10 + 0.2 = \underline{\underline{10.2}}$ $S(12) \approx S(11) + 0.2 = \underline{\underline{10.4}}$

Sect 3.4

#3 $y = x^9 \quad \frac{dy}{dx} = \underline{\underline{9x^8}}$

#27 $f(t) = 2t^2 - 3t + 1 \quad f'(t) = 2(2t) - 3 = \underline{\underline{4t - 3}}$

#43 $h(t) = \frac{3}{t^{3/5}} - \frac{6}{t^2} = 3t^{-3/5} - 6t^{-2}$
 $h'(t) = 3(-\frac{3}{5}t^{-8/5}) - 6(-\frac{1}{2}t^{-3/2}) = \underline{\underline{-\frac{9}{5t^{8/5}} + \frac{3}{t^{3/2}}}}$

#83 $N(x) = 1000 - \frac{3,780}{x}$ for $5 \leq x \leq 30$

$N(x) = 1000 - 3780x^{-1}$

(A) $N'(x) = -3780(-1x^{-2}) = \frac{3780}{x^2}$

(B) $N'(10) = \frac{3780}{10^2} = 37.8$ growth rate for sales 37.8 for each additional 1,000 on ads at $x=10$

$N'(20) = \frac{3780}{20^2} = 9.45$ growth rate slows down at $x = \$20,000$

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Sect 3.5

#9 $f(x) = (x^2 + 1)(2x - 3) \quad f'(x) = (2x)(2x - 3) + (x^2 + 1)(2) = 6x^2 - 6x + 2$

#14 $f(x) = \frac{3x+5}{x^2-3} \quad f'(x) = \frac{(x^2-3)(3) - (3x+5)(2x)}{(x^2-3)^2} \quad \left(\text{or } \frac{-3x^2 - 10x - 9}{(x^2-3)^2}\right)$

#67 $x = \frac{4,000}{0.1p+1} \quad 10 \leq p \leq 70$ demand

(A) $\frac{dx}{dp} = \frac{(0.1p+1)(0) - 4000(0.1)}{(0.1p+1)^2} = \frac{-400}{(0.1p+1)^2}$

(B) $x(40) = \frac{4000}{4+1} = 800 \quad x'(40) = \frac{-400}{(4+1)^2} = -16$ drops by 16 for each additional \$1

(C) est. $x(41) \approx 800 - 16 = \underline{\underline{784}}$

#69 $C(t) = \frac{0.14t}{t^2+1}$ (A) $C'(t) = \frac{(t^2+1)(0.14) - 0.14t(2t)}{(t^2+1)^2} = \frac{-0.14t^2 + 0.14}{(t^2+1)^2}$

(B) (note: $C'(1) = 0$ max reached)

$C'(0.5) = \frac{0.105}{(1.25)^2} = 0.0672$ conc. incr.

$C'(3) = \frac{-1.12}{(10)^2} = -0.0112$ concn. decr. slowly

Sect 3.6

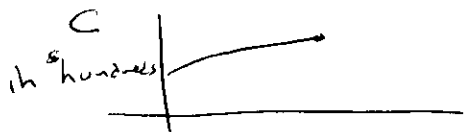
#3 $\frac{d}{dx} (4-2x^2)^3 = 3(4-2x^2)^2 (-4x)$

#15 $f(x) = (x^3 - 2x^2 + 2)^8$ $f'(x) = 8(x^3 - 2x^2 + 2)^7 (3x^2 - 4x) = (24x^2 - 32x)(x^3 - 2x^2 + 2)^7$

#23 $f(x) = (4x-3)^{1/2}$ $f'(x) = \frac{1}{2}(4x-3)^{-1/2} \cdot 4 = \frac{2}{\sqrt{4x-3}}$ ($\neq 0$ for any x)
 $f'(3) = \frac{2}{\sqrt{12-3}} = \frac{2}{\sqrt{9}} = \frac{2}{3}$ $f(3) = \sqrt{9} = 3$ $y-3 = \frac{2}{3}(x-3)$
 or $y = \frac{2}{3}x + 1$

#45 $f(x) = x(4-x)^3$ $x=2$ $f(2) = 2 \cdot 2^3 = 16$
 $f'(x) = 1 \cdot (4-x)^3 + x \cdot 3(4-x)^2(-1)$ $\Big|_{x=2} = 1 \cdot 8 + 2 \cdot 3(2)(-1) = -16$
 $y-16 = -16(x-2) \Rightarrow y = -16x + 48$

#75 $C(x) = 10 + \sqrt{2x+16}$ $0 \leq x \leq 50$ (A) $C'(x) = 0 + \frac{1}{2}(2x+16)^{-1/2} \cdot 2 = \frac{1}{\sqrt{2x+16}}$



(B) $C'(24) = \frac{1}{\sqrt{64}} = \frac{1}{8} = 0.125$ extra/marginal cost at 24 is \$12.50
 $C'(42) = \frac{1}{\sqrt{100}} = \frac{1}{10} = 0.100$ and at 42 is \$10

Sect 3.7

#7 $P(x) = 5x - 0.005x^2 - 450$ $0 \leq x \leq 1,000$
 $P'(x) = 5 - 0.01x$
 (A) $P'(450) = 5 - 4.5 = 0.5$ profit incr. by \$0.50 for each new cassette
 $P'(750) = 5 - 7.5 = -2.5$ profit decr. by \$2.50 " " " "

#15 $\frac{P}{\$16}$ $\frac{x}{200}$ $C = 1400 + 4x$
 $\frac{14}{300}$

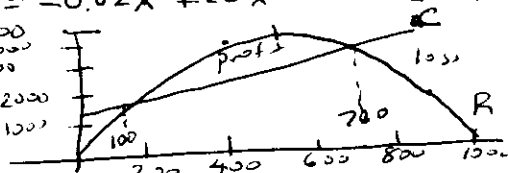
(A) $m = \frac{14-16}{300-200} = -\frac{2}{100} = -0.02$ $P = -0.02x + b$
 $16 = -0.02(200) + b \Rightarrow b = 20$ $P = -0.02x + 20$

(B) $R = xp = x(-0.02x + 20) = -0.02x^2 + 20x$ $0 \leq x \leq 1000$ (otherwise $p < 0$)

(C) $C = 1400 + 4x$

(D) \Rightarrow graph

(E) $P = R - C = -0.02x^2 + 20x - (1400 + 4x)$
 $= -0.02x^2 + 16x - 1400$



(F) $P'(x) = -0.04x + 16$

$P'(250) = +6$, $P'(775) = -3$ incr. at 250, decr. at 775

#19 $p = 20 - \sqrt{x}$ (A) $R(x) = x(20 - \sqrt{x}) = 20x - x^{3/2}$
 $C(x) = 500 + 2x$ (B) $C(x) = 500 + 2x$

Using Derive or graphics calculator

$x = 43.98 \approx 44$ is 1st intersection
 and $x = 257.98 \approx 258$ is 2nd intersection

