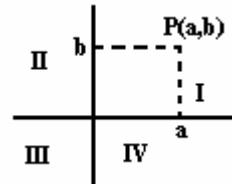
Brief Summary of Chapter 1 – College Mathematics – Prof. Richard B. Goldstein

1.1 Functions

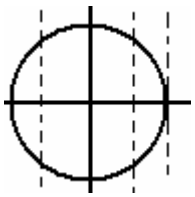
Cartesian Coordinate System 

Graphing pt. by pt. | x | y |

Function - a rule that produces a correspondence between two sets of elements such that to <u>each</u> element in the <u>first set</u> there corresponds <u>one and only one</u> element in the <u>second set</u>

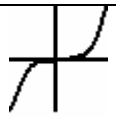
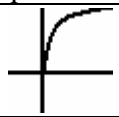
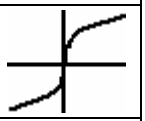
Domain - set of allowable points of $y = f(x) : x \in \mathcal{D}$ (no divisions by 0, neg. sq. roots)

Range - second set of points – values that y obtains ex. $Y \in [0, \infty)$

Vertical Line Test  but  is called a **<u>relationship</u>**

$y = x^2 + 4$ (OK) $x^2 + y^2 = 4$

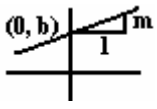
1.2 Elementary Functions; Transforms

function	x	|x|	x^2	x^3	$\sqrt{x}$	$\sqrt[3]{x}$
name	identity	absolute value	square	cube	square root	cube root
graph						
Domain	R	R	R	R	$[0, \infty)$	R
Range	R	$[0, \infty)$	$[0, \infty)$	R	$[0, \infty)$	R

Shifts:

Vertical	$y = f(x) \pm k$	up or down
Horizontal	$y = f(x \pm h)$	left or right
Reflection	$y = -f(x)$	
Vert. expansion/contraction	$y = Af(x)$	(see handout)

1.3 Linear Functions

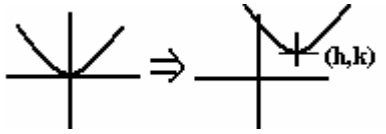
$y = mx + b$ also $Ax + By = C$ 

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $y - y_1 = m(x - x_1)$

1.4 Quadratic Functions

$y = ax^2 + bx + c = a(x - h)^2 + k$ where $h = -\dfrac{b}{2a}$, $k = c - ah^2$

vertex (h, k) roots $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$



Brief Summary of Chapter 2 – College Mathematics

2.1 Polynomials and Rational Polynomials

polynomial n^{th} degree $\quad f(x) = P_n(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$

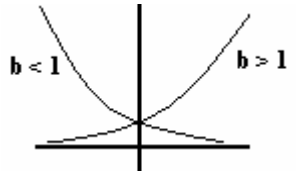
rational polynomial $\quad f(x) = \dfrac{P_n(x)}{Q_m(x)} = \dfrac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{n-1} + \cdots + b_1 x + b_0}$

asymptotes – vertical at $x = c$ where $Q_m(c) = 0$

horizontal – divide by largest power of x and let $x \to \infty$

2.2 Exponential Functions

$f(x) = b^x \qquad b > 0,\ b \neq 1 \qquad b$ = base



$\mathcal{D}$ = R, Range = $(0, \infty)$

$a^x a^y = a^{x+y},\ (a^x)^y = a^{xy},\ a^x = a^y \Rightarrow x = y$

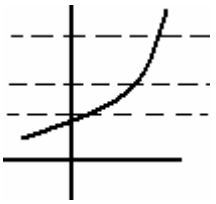
base e = 2.71828…

examples: growth: $N_0 e^{kt}$

decay: $N_0 e^{-kt}$

continuous compound interest: $A = Pe^{rt}$

2.3 Logarithm



1 – 1 Function horizontal lines cross only once and each range value corresponds to exactly <u>one</u> domain value

Inverse $\quad y = f(x) \Rightarrow x = f^{-1}(y)$

$y = \log_b x \Leftrightarrow x = b^y$

$\log x = \log_{10} x$

$\ln x = \log_e x$

$\log_b x = \dfrac{\ln x}{\ln b}$

properties

$\log(AB) = \log A + \log B$

$\log(A / B) = \log A - \log B$

$\log(A^B) = B \log A$