Limit Laws - **Prof. Richard B. Goldstein**

Suppose that *c* is a constant and the limits: $\lim\limits_{x\to a} f(x)$ and $\lim\limits_{x\to a} g(x)$ exist. Then,

[1] $\lim\limits_{x\to a}\left[f(x)+g(x)\right] = \lim\limits_{x\to a} f(x) + \lim\limits_{x\to a} g(x)$

[2] $\lim\limits_{x\to a}\left[f(x)-g(x)\right] = \lim\limits_{x\to a} f(x) - \lim\limits_{x\to a} g(x)$

[3] $\lim\limits_{x\to a}\left[cf(x)\right] = c\lim\limits_{x\to a} f(x)$

[4] $\lim\limits_{x\to a}\left[f(x)g(x)\right] = \lim\limits_{x\to a} f(x) \bullet \lim\limits_{x\to a} g(x)$

[5] $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)}$ if $\lim\limits_{x\to a} g(x) \neq 0$

[6] $\lim\limits_{x\to a}\left[f(x)\right]^n = \left[\lim\limits_{x\to a} f(x)\right]^n$ where n is a positive integer

[7] $\lim\limits_{x\to a} c = c$

[8] $\lim\limits_{x\to a} x = a$

[9] $\lim\limits_{x\to a} x^n = a^n$ where n is a positive integer

[10] $\lim\limits_{x\to a}\sqrt[n]{x} = \sqrt[n]{a}$ where n is a positive integer (If n is even, we assume a > 0)

[11] $\lim\limits_{x\to a}\sqrt[n]{f(x)} = \sqrt[n]{\lim\limits_{x\to a} f(x)}$ where n is a positive integer

(If n is even, we assume $\lim\limits_{x\to a} f(x) > 0$)

Theorem: If f is a polynomial or a rational polynomial and *a* is in the domain of f, then $\lim\limits_{x\to a} f(x) = f(a)$

Theorem: $\lim\limits_{x\to a} f(x) = L$ if and only if $\lim\limits_{x\to a^-} f(x) = L = \lim\limits_{x\to a^+} f(x)$

Replacement Theorem: If f(x) = g(x) for all x in the domain of both except perhaps at x = a and the limit: $\lim\limits_{x\to a} g(x) = L$ exists, then $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} g(x) = L$

Squeeze Theorem: If $f(x) \leq g(x) \leq h(x)$ where *x* is near *a* (except possibly at *a*) and $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} h(x) = L$ then $\lim\limits_{x\to a} g(x) = L$